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The control volume concept in aeronautical engineering

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THE CONTROL VOLUME CONCEPT IN AERONAUTICAL ENGINEERING

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THESIS

THE CONTROL VOLUME CONCEPT

IN AERONAUTICAL ENGINEERING

by

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Thesis Advisor: O. Biblarz

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The Control Volume Concept
in Aeronautical Engineering

by

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ABSTRACT

This thesis contains a handout covering the Fundamental Physical Laws (Continuity, Momentum, and Energy) used in Aeronautical Engineering which are transformed from the control mass or system form into the control volume form. It is intended that this handout serve as a self-studying guide for students in the core of the Aeronautical Engineering Program at the Naval Postgraduate School and as a reference during the graduate level courses.

TABLE OF CONTENTS

I. INTRODUCTION

A. THE PROBLEM UNDER CONSIDERATION

In the Aeronautical Engineering Curricula at the Naval Postgraduate School the development of expressions for the Fundamental Physical Laws (Continuity, Momentum, and Energy) is covered at different depths in courses such as Basic Fluid Dynamics, Basic Thermodynamics and Gas Dynamics during the core.

Due to the lack of time, these topics are usually treated in an introductory fashion and the notes taken by the students are necessarily incomplete with some repetition between the various courses.

Also, in some cases the development of these equations is done without a systematical recognition that control-volume approach is used in opposition to the system or fixed-mass approach (because of the inconvenience of the latter for problems related to fluids). In addition to the problem previously described, it happens sometimes that the student is not aware that the equation the professor is using is a "disguised" form of one of the Fundamental Physical Laws for which he may already have some intuition.

The situation may be depicted in the following chart:

TOPIC:	FUNDAMENTAL PHYSICAL LAWS INTEGRAL FORMS	FUNDAMENTAL PHYSICAL LAWS DIFFERENTIAL FORMS	PARTICULAR FORMS NAVIER-STOKES, ETC.
DEVELOPED AND USED IN COURSES:	2041, 2042	3043	3043, 4431, 4432 4501, 4502, 4511 4512
STAGE IN THE PROGRAM:	CORE		GRADUATE
OBJECTIVES:	← CREDIBILITY AND RIGOR →	← CONTINUITY OF SUBJECT MATTER →	
	← HANDINESS →		

B. TYPE OF STUDENTS ATTENDING THE CORE

A considerable number of students taking the sequence of core courses are coming from the fleet after a number of years (between four and eight), since the completion of undergraduate education in Science or Engineering.

It can be said that sufficient background exists in most of the cases but it is buried under several years of performance of nonacademic duties.

The most noticeable effect of these inactive years is the difficulty in dealing with abstract concepts such as mathematical symbols and models intended to depict a situation of engineering interest (the control volume, for example).

C. A PROPOSED SOLUTION FROM A STUDENT STANDPOINT

The author believes that a handout received by the students before or during the core will provide them with a set of notes

for self-study during the core and for future reference in the graduate sequence of courses.

The proposed handout on the Fundamental Physical Laws is a version of material covered in many books, some of them being more rigorous and elegant but often not appealing to the students.

What is it that bothers a student in this situation when the topic is treated in class or in his textbooks?

First of all, the terminology. The reader must be warned that the author is perhaps strongly biased in this respect because English is not his native language. But even students without this problem sometimes are confused by words or do not know the exact meaning of some terms in Thermodynamics or Fluid Dynamics. For example, look up the word "steady" in a dictionary. Reference 1 gives the following meanings for it: firm, stable, not shaky, regular, uniform, continuous, constant in behavior, etc. Assuming that the student knows all these meanings, if the word "steady" is used without rigorously defining it in a scientific context (i.e., partial derivatives with respect to time are zero), a sensation of ambiguity will remain and no convenient abstraction can be expected to last in the student's mind.

Important terms must be carefully defined from the beginning and the other terms introduced as needed during the development. The use of these terms must be shown by means of examples and the degree of retention by the student must be frequently tested. (Notice how the term system is treated in the text.)

A second source of trouble is the prolific contribution to Science by men such as Newton, Euler, Bernoulli (three of them! Jacob (1664-1705); Johan (1667-1748); and Daniel (1700-1782)); Laplace and many others. When someone quotes "Newton's Law," to which one is he referring – the First, Second, or Third? Maybe it is Newton's Law of Friction. Sometimes the student is taking, simultaneously, a course in Mathematics where the same names are also quoted.

The fundamental points for a student in the core in this respect are:

1. Not to get confused by the terminology and particular names.

2. To recognize where an equation originates; what is the significance of the various terms; and what are the constraints and assumptions for its derivation.

3. To make actual use of the equations.

Another problem arises from Algebra. It is not that this branch of Mathematics is obscure or difficult for a student at this stage of his education, but that it is simply uninteresting.

Oftentimes during a long and tedious derivation of an equation the student's mind wanders and the sensation of "magic" results when a final and neat expression appears. The following corrective actions may be proposed:

1. State from the beginning the starting and ending points of the derivation.

2. Underline the milestones in the derivation.

3. Ask questions of the student in the middle of the derivation to keep him interested or simply awake.

4. Skip on purpose some steps but ask the students to go through them for completeness.

5. Use the most compact notation available but only after being sure that the notation is properly introduced and that this greater degree of abstraction is consistent with the level of the student.

Perhaps many of these problems are avoided by using a textbook writing technique called "Programmed Instruction." The opinion of the author is that writing a handout fully using this technique is out of the scope of a Master's Degree Thesis, but some of the features of this technique can be incorporated and the possibilities of improvement in this respect are unlimited.

D. THE VARIABLE OBJECTIVES OF THE HANDOUT

The objective of the proposed handout is not necessarily to teach a technique or a skill (such as the use of the Laplace Transform to solve certain differential equations). The objectives of this handout are variable from the student standpoint, accordingly to what stage of his education he is at.

At the beginning, during the core, the main text of the handout is intended to satisfy the objective of CREDIBILITY and RIGOR. The student should be familiar (and if he has forgotten, the very first part of the handout will serve as a refresher) with the system or fixed-mass approach. The handout will lead him neatly and rigorously to the control volume approach through a transformation or extension of the system approach.

This needs to be done only once and it can be done using a generalized notation. Also it is believed that a self-study text has the potential of removing the usual class-room problems.

In the next steps the student is led through specializations of the control volume formulation. The students who are in the core are not expected to carry out these further specializations, but he is expected to do so in his graduate courses. Here, the objective of the handout is to provide CONTINUITY OF SUBJECT MATTER through the whole sequence of courses.

The same objective is pursued in the extensions leading to the differential form of the Fundamental Physical Laws and to other particular forms. He is not expected to carry out these derivations, but he is expected to understand where the equations come from and (eventually) the use of them in the context of the graduate level courses. Here the "mystery" of the equation written down without proof might be removed and the student uneasiness with an advanced subject alleviated.

The entire handout should serve properly as a review and reference during all the stages of his career at the Naval Postgraduate School. The objective in this respect is HANDINESS. This is perhaps the most ambitious aim of this work.

E. THE LEARNING PROCESS

This thesis has been written according to the following basic ideas with respect to the learning process:

1. The assumption that the student is motivated; he has the desire to learn.

2. No mathematical background is assumed beyond undergraduate level.
3. Certain basic vocabulary is assumed but the important concepts are defined or explained carefully and their understanding emphasized and tested in several parts of the text.
4. There would be systematic reading of the context during the self-study use of these notes. The student will not proceed to another section without a fair understanding of the previous. (Approximate relative times for reading are stated.)

Along the text, several quizzes, programmed questions and exercises are inserted to help the student recognize his own degree of retention of what he was supposed to learn.

When a new quantity is introduced and assigned a symbol, the corresponding units are shown in the English system only as an example to help the reader recognize and associate the nature of the quantity.

Also, to some restricted extent, some provisions are made in order to allow the student to extend his recently obtained knowledge to new situations.

F. SUMMARY OF THE PRESENTATION OF THE SUBJECT IN THE HANDOUT

The general idea followed during the development of the thesis was to treat the topic at the highest degree of generality and compactness consistent with the level of the student and then to particularize to arrive at useful forms of the equations.

It can be mentioned in this respect that the reduction of the expressions of the three Fundamental Physical Laws to one using a generalized notation, was based on Ref. 2 after consulting several undergraduate level books and notes [Refs. 3 through 11].

The concepts involved in the control volume approach were applied to this generalized expression of the Fundamental Physical Laws for a system. Later a general expression of the Fundamental Physical Laws for a control volume was developed. Then this expression was specialized to the particular laws in integral form and later in differential form.

The advantage of this method is that the mathematical transformation needs only to be done once, enabling the student to concentrate on the actual mathematics without having to identify the "proof" with a single Fundamental Physical Law.

The steps in particularization of the equations are contained in the supplements where the Navier-Stokes, Bernoulli and Euler equations are developed having in mind that very often a fair knowledge of these equations is required in courses such as Boundary Layer Theory, Convective Heat and Mass Transfer, Magnetohydrodynamics, Advanced Gas Dynamics, Turbomachinery and Aerodynamics of Wings and Bodies.

G. THE SCOPE OF THE APPROACH CHOSEN

This thesis is a student's point of view on the subject, taking into consideration the difficulties found during his education.

This type of approach may be helpful to fellow students in the same way as it has been for the author.

Under the title of this thesis, a broader range of subjects can be covered. However, some amount of limitation was found to be necessary in order to fulfill most of the purposes enunciated earlier in this Introduction.

It is necessary to mention that the treatment of the Equation of State which is complementary to the Fundamental Physical Laws has been omitted. The reason for this is that this topic is covered in a thesis written at the same time by another student [Ref. 12].

II. THE HANDOUT

In order to have the proposed handout in a style and format more useful for its purpose, it is inserted in the Appendix.

III. A SURVEY OF THE USE OF THE HANDOUT BY STUDENTS

An earlier version of the handout was distributed to test its acceptance by the students.

This was done in Course AE 2041, Basic Thermodynamics, where Ref. 6 was used as a textbook. Upon the completion of the Course the instructor made a survey of the opinion of the students about the handout. The following are some results of such a poll:

95 % read the handout in its entirety.

88 % had sufficient background to understand the material.

95 % felt that the material was of sufficient difficulty to merit a handout.

95 % felt that the material was presented in a logical order.

When asked whether they would prefer their textbook, the handout, neither, or both, the response was:

40 % present text

35 % handout

15 % both

10 % neither

One important conclusion based on the survey is that at least the handout received was a desired instructional aid.

With the improvements made since then and other improvements that can be made after testing it again, the handout may be transformed in a very useful instructional aid to be received by the students at the beginning of their career at the school.

IV. RECOMMENDATIONS FOR FUTURE WORK

The handout may be improved mainly in the supplements.

In Supplement C, the differential form of the Energy Equation is only stated, suggesting that its development is similar to the development of the Navier-Stokes Equations which in turn are not derived in detail for the sake of conciseness.

Perhaps a few more steps may be added in the development of the Navier-Stokes Equations. The complete derivation of the Energy Equation in differential form is needed to satisfy the requirement of completeness for reference use.

There are also opportunities for improvement in the main body in the style of the presentation. After being used by the students, the programmed characteristics may be increased to overcome difficult steps.

APPENDIX

Naval Postgraduate School
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THE CONTROL VOLUME CONCEPT
IN AERONAUTICAL ENGINEERING

From an M.S. Ae. E. Thesis

By: LT Carlos Tromben

Thesis Advisor:
Asst Professor Oscar Biblarz
1971

INDEX

TABLE OF SYMBOLS AND ABBREVIATIONS

A	area on the $\hat{c}$ontrol $\hat{s}$urface
CV	$\hat{c}$ontrol $\hat{v}$olume
CS	$\hat{c}$ontrol $\hat{s}$urface
E	energy
e	energy per unit mass
g	acceleration of gravity
g_c	dimensional constant in Newton's 2nd Law
h	enthalpy per unit mass
h_t	total enthalpy per unit mass
k	thermal conductivity
m	mass
$\vec{M}$	moment
$\vec{n}$	unit normal vector
$\vec{P}$	momentum
$\mathbb{P}$	rate of work
p	pressure
Q	heat
$\dot{Q}$	heat flow. Heat per unit time.
$\dot{q}$	heat flow per unit area. Heat per unit time per unit area.
$\vec{r}$	position vector of the center of mass of a system of particles or position vector of a particle.
S	entropy
s	entropy per unit mass
syst	system
T	temperature
t	time

U	internal energy
V	volume
$\vec{v}$	velocity. ($v = \lvert\vec{v}\rvert$ speed)
W	work
X	an extensive property that is "conserved" within the $\hat{s}$ystem
Y	a quantity that represents how the $\hat{s}$ystem interacts with the $\hat{s}$urroundings
z	height above a datum point
μ	viscosity
$\vec{\zeta}$	arbitrary vector
ρ	density, mass per unit volume
Φ	dissipation function

I. PRELIMINARIES

A. A SUGGESTION ON HOW TO USE THIS HANDOUT

This handout is supposed to promote a better understanding of the Fundamental Physical Laws (Continuity, Momentum, and Energy) and is to be read out of the classroom (i.e., self-study).

To obtain better results, (usually) at the beginning of each chapter or section there is a note suggesting how far you, the student, should go on reading continuously. We do not forecast how long it will take to read and understand each unit because of individual differences, but our guess is that the first two units will take you about one hour. The estimates on the following sections are indicated relative to the first.

If you do not have the time now to read continuously as suggested, try it some other time, because partial reading will not benefit you in any way.

Also, there are <u>three</u> types of testing material inserted along the text and you should understand their purposes:

1. <u>Self-Check Quizzes</u>

The purpose of these quizzes is to help you recognize your own degree of retention before proceeding to the next page of the material. Be sure to correct the wrong answers, and do not continue unless you feel you understand the difficulties you had. The answer is usually at the bottom of the page, written upside down.

2. <u>Programmed Questions</u>

The primary purpose of these questions is to remind you of important points presented in previous sections. You must fill the blank space left on purpose and the answer is usually at the bottom of the page, written upside down.

3. <u>Exercises</u>

These are not truly testing material. Their purpose is to make you think about further implications of the subject in an open-ended fashion. No answer is given for these.

B. FUNDAMENTAL PHYSICAL LAWS

In the solution of problems in Aeronautical Engineering, concepts from the Physical Sciences called "Fundamental Physical Laws," must be applied. These laws relate to mass, momentum, and energy and are commonly referred to as "conservation laws" because under many circumstances they are truly conserved; however, the word conservation in its common usage takes a broader meaning which may be stated as "accounted for" or "not destroyed." We will use the term "Fundamental Physical Laws (FUPLAs)" because this will permit us to include the second law of thermodynamics relating to the entropy which is not conserved in a natural process. In addition to the FUPLAs, information is needed about the fluid itself usually contained in the Equation of State. This equation describes a set of unique relationships between state properties and may be found in algebraic form (the ideal gas law), tabular form (the steam tables), or graphical form (p-v-T surfaces, etc.).

C. BASIC DEFINITIONS

Read continuously from here to the end of Section D.

1. A <u>Word about the Notation</u>

Through all this work a ^ (caret) will be used over the first letter of a word which has a specific meaning in this text. For example, whenever the word cycle appears with a ^ over the letter "c" (i.e., ĉycle), means a precise concept and that there is no other meaning for the same word.

All words beginning with a letter accented with ^ are carefully defined in the following list.

Fluid: a substance that deforms continuously under the action of a shear stress.

System: a collection of matter of fixed identify (Control Mass).

Control Volume (CV): a region in space of fixed size through which fluid flows. It may be stationary, moving uniformly (inertial), or accelerating (non-inertial). In this work we will consider the CV always fixed with respect to an inertial coordinate axis system,[1] unless otherwise specified. This volume is bounded by a control surface.

We want to insist on the idea that, although the control volume has to be fixed with respect to an inertial system of coordinates, it does not necessarily need to be fixed in space. The following illustration shows this. The vehicle may be moving through space but the control volume is fixed with respect to the vehicle.

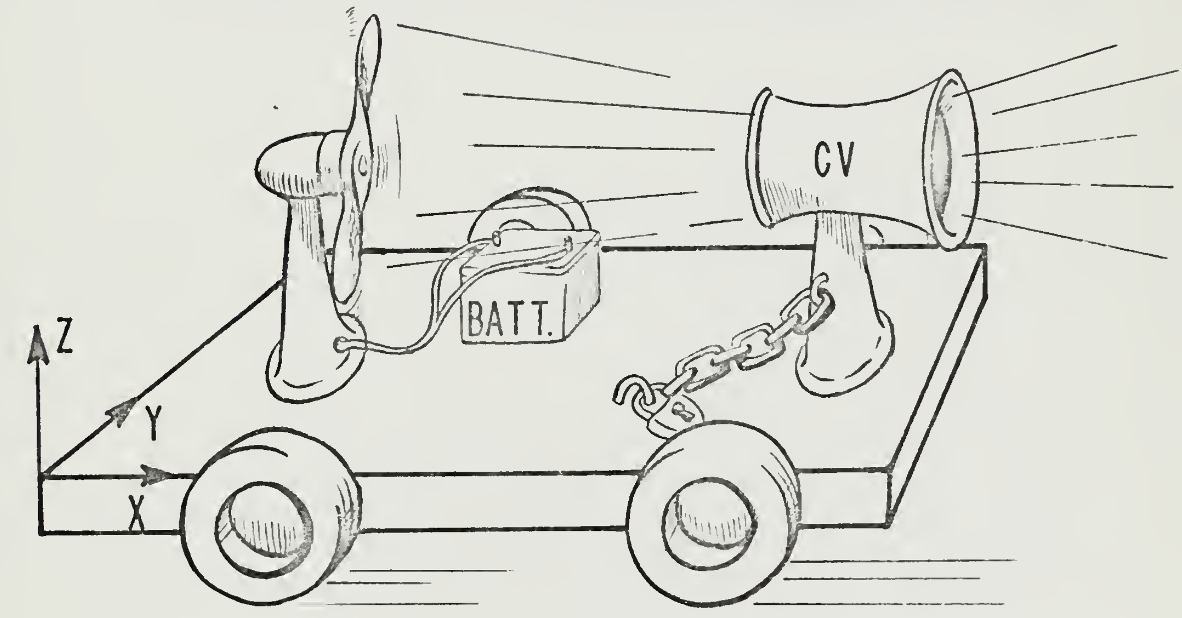

Figure 1. Fixed Control Volume with respect to inertial frame of reference.

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[1] Our development is constrained to this case for simplicity and to fit into the scope of the handout like this. Important Aeronautical Engineering applications require non-inertial frames of reference, such as in turbomachinery, for example, [Ref, 13].

$\hat{S}$urroundings: everything outside the $\hat{s}$ystem

$\hat{E}$xtensive $\hat{P}$roperty: a $\hat{p}$roperty that is mass dependent. It can be a scalar, as energy or a vector, as momentum

$\hat{F}$undamental $\hat{P}$hysical $\hat{L}$aws (FUPLAs): there are three fundamental physical laws which, with the exception of relativistic and nuclear phenomena, apply to each and every flow, independently of the nature of the $\hat{f}$luid under consideration

$\hat{S}$tate: the condition of the $\hat{s}$ystem characterized by the values of its properties

$\hat{P}$rocess: the path of the succession of states through which the $\hat{s}$ystem passes

$\hat{C}$ycle: a $\hat{s}$ystem that undergoes a series of $\hat{p}$rocesses and always returns to its initial state is said to have gone through a $\hat{c}$ycle

$\hat{P}$roperties: characteristics of a $\hat{s}$ystem that define its state.

TABLE I

FUPLAs (From Observation)	MATHEMATICAL FORMULATION
Conservation of Mass	Continuity Equation
Newton's Second Law of Motion	Momentum Equation
First Law of Thermodynamics	Energy Equation

Note that Table I is incomplete. Other FUPLAs will be seen later.

2. <u>A Word about FUPLAs</u>

Perhaps the reader has become uneasy about the previous wording. All those terms, with the caret (which denotes a specific meaning) and the solemn title "$\hat{F}$undamental $\hat{P}$hysical $\hat{L}$aws," may lead one to think that these laws have something mysterious or difficult to understand.

This should not be so.

These laws were deduced from observations of nature. Nature behaves this way. It has always behaved so and nobody has reliably observed a contradiction to this behavior.

This is the only "proof" of these laws.

Later these $\hat{F}$undamental $\hat{P}$hysical $\hat{L}$aws are put into a mathematical form in order to manipulate and work with these concepts in the precise and synthetic language of the mathematical symbols.

This almost trivial example shows that the FUPLAs belong to everyday experience: If you had a dozen apples which you were to squeeze to make apple sauce with, you would not be conserving the number of apples. What is to be

accounted for is the matter (mass) when the seeds and other discards are taken into account.

FUPLAs are the generalization of many reliable observations (i.e., matter is not destroyed, it is transformed into other forms of matter or into energy).

D. SELF-CHECK QUIZ NO. 1

Select the proper answer

1. The Ĉontrol V̂olume is:

   a. a collection of mass of fixed identity
   b. always the volume occupied by the ŝystem
   c. a region in space of fixed size through which fluid flows
   d. always stationary

2. The F̂undamental P̂hysical L̂aws:

   a. are deduced from observation of nature
   b. can the proven using highly complicated mathematical procedures and as a result of this, most of the textbooks do not have them
   c. apply to certain types of f̂luids, flowing under special conditions
   d. have no mathematical expressions

Answers: 1(c) and 2(a)

EXERCISES:

What is conserved when an object of mass m falls from a height h in a gravity field?

A typical reference ŝtate for air is standard conditions. Do you know what this means?

II. D̂EVELOPMENT OF EXPRESSIONS FOR THE F̂UNDAMENTAL P̂HYSICAL L̂AWS FOR A ŜYSTEM

Read continuously up to the end of Section E.

A. CONSERVATION OF MASS

A ŝystem is ______________________________

Let us consider the following system

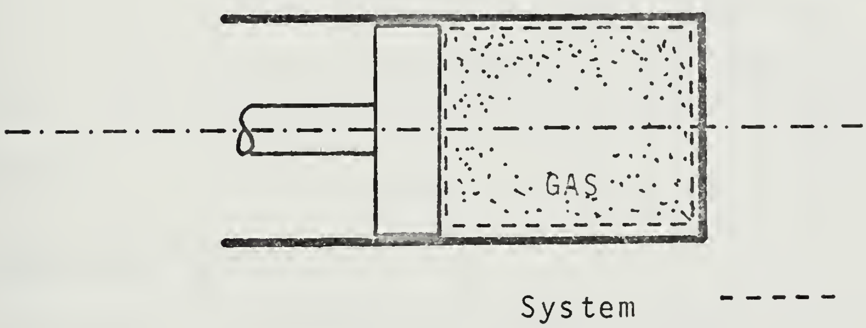

Figure 2. Gas in a closed cylinder

Experience tells us that matter will not flow in or out through the solid boundaries of this ŝystem (or any other), as time elapses or as the identified ŝystem wanders through space. Also, matter will not be created spontaneously within the ŝystem. From this last statement we may write

$$m_{syst} = \text{constant} \tag{1}$$

$$m_{syst} = \int_{syst} dm \tag{2}$$

Answer: a quantity of mass of fixed identity.

If $\rho = \frac{dm}{dV}$, then $m_{syst} = \int_{syst} \rho dV$ (3)

Differentiating with respect to time in Eq. 3 and keeping in mind that the left-hand side is a constant (from Eq. 1) we get

$$\frac{d}{dt}\int_{syst} \rho dV = 0 \qquad (4)$$

Note that in the above, as well as throughout the rest of this Section II, integrals over the mass (as in Eq. 2) are being used. They are changed into volume integrals provided we can use the mass density ρ. In other words, for the system analysis, integrals over a fixed quantity of <u>mass</u> are being dealt with, and eventually these basic laws in terms of ĉontrol v̂olume will be expressed. In order to avoid confusion as to which one the volume integral belongs, the term "syst" will appear by the integral sign when a fixed mass is being analyzed and the term "CV" when a fixed or control volume is being analyzed.

B. MOMENTUM EQUATION

Consider a differential volume element of our ŝystem which is traveling at velocity $\vec{v}$. This volume element will have a mass dm and again

$$dm = \rho dV \qquad (5)$$

and a momentum

$$d\vec{P} = \vec{v} dm \qquad (6)$$

$$d\vec{P} = \vec{v} \rho dV \qquad (7)$$

So that the total momentum for the ŝystem will be

$$\int_{syst} d\vec{P} = \int_{syst} \vec{v} \rho dV \qquad (8)$$

Differentiating Eq. 8 with respect to time

$$\frac{d}{dt}\int_{syst} d\vec{P} = \frac{d}{dt}\int_{syst} \vec{v}\rho dV \qquad (9)$$

The Momentum equation is the expression of Newton's Second Law of Motion which states:

> "The time rate of change of momentum of a $\hat{s}$ystem is proportional to the net external force acting on system and takes place in the direction of the net force."

(Is there a "proof" for this law?)

Therefore,

$$\vec{F}_{net} = \frac{d}{dt}\int_{syst} d\vec{P} \qquad (10)$$

The net force is made up of contributions of two kinds of force, namely,

$$\vec{F}_{net} \equiv \sum \vec{F}_{volume} + \sum \vec{F}_{surface} \qquad (11)$$

The above distinction regarding the net force is needed because of the different nature between, say, the force of gravity and a pressure force (Supplement A expands on the nature of these forces).

Inserting Eq. 9 into Eq. 10:

$$\vec{F}_{net} = \frac{d}{dt}\int_{syst} \rho\vec{v}\, dV \qquad (12)$$

If there is no net external force, Eq. 8 becomes

$$\frac{d}{dt}\int_{syst} d\vec{P} = 0 \qquad (13)$$

or

$$\vec{P} = \text{constant (i.e., momentum is conserved)} \qquad (14)$$

The student should eventually learn to recognize the difference between the collision of two billiard balls and of two balls of putty in terms of the momentum equation.

Again, all that has been considered so far, with respect to the Conservation of Momentum, is restricted to the system approach. At this point the above will not be very useful if the ŝystem is difficult to identify.

C. ENERGY EQUATION

The Energy Equation is the mathematical expression of the First Law of Thermodynamics, which can be stated as follows: "If a ŝystem is carried through a cycle, the total heat added to the ŝystem from its ŝurroundings is proportional to the work done by the ŝystem on its ŝurroundings."

Let us have a ŝystem, that is, a quantity of matter of fixed identity, traveling through space (changing its position with respect to an inertial frame of reference as time elapses).

Let the path of the ŝystem be in such a way that it will return to its initial ŝtate. In doing so, it has performed a ĉycle.

During the ĉycle, heat may be added to the ŝystem by its ŝurroundings and in turn, work may be done by the ŝystem on the ŝurroundings. A useful ĉycle is depicted in Figure 3.

Notice that the previous discussion and Figure 3 refers to a system undergoing a cycle, that is, ______________________ ______________________. A similar discussion may be made for a system undergoing a single p̂rocess.

Since energy is related to heat and work, for a process — an expression for the First Law of Thermodynamics in the rate form can be written:

$$\left(\begin{array}{l}\text{Rate of heat}\\ \text{flow into the}\\ \text{system}\end{array}\right) - \left(\begin{array}{l}\text{rate of work}\\ \text{done by system}\\ \text{to the}\\ \text{surroundings}\end{array}\right) = \left(\begin{array}{l}\text{rate of change of}\\ \text{energy within the}\\ \text{system}\end{array}\right) \qquad (15)$$

Here, the following sign convention is used: Heat <u>added to</u> the ŝystem by the ŝurroundings, positive (+). Work <u>done on</u> the ŝurroundings by the ŝystem, positive (+).

Answer: a Series of processes that always returns the ŝtate of the ŝystem to its original ŝtate.

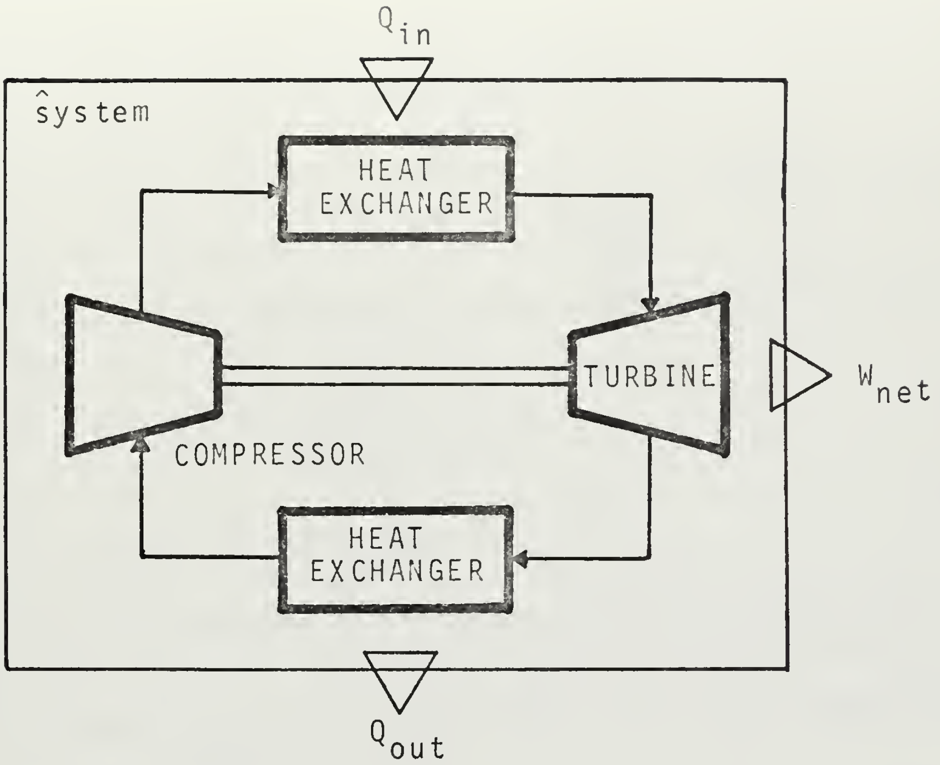

Figure 3. Gas cycle

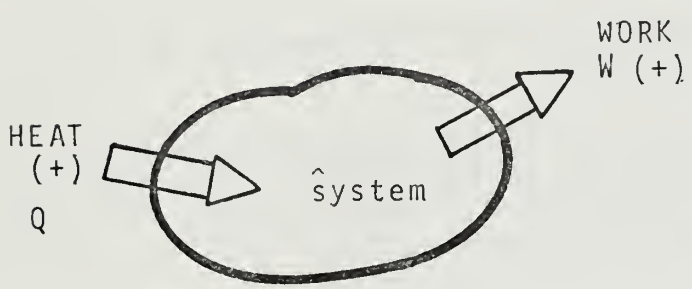

Figure 4. Sign convention for the First Law

This sign convention is not universal. Some authors use other conventions.

Let us define the following symbols:

$\dot{Q}$ rate of heat flow (e.g., in $\frac{Btu}{sec}$)

$\mathbb{P}$ rate of work done (e.g., power i.e., in $\frac{ft\text{-}lbf}{sec}$ or watts)

e total energy per unit mass (e.g., $\frac{Btu}{lbm}$)

Then, one may write the energy equation for a system from Eq. 15:

$$\dot{Q} - \mathbb{P} = \frac{d}{dt}\int_{syst} e \, dm \tag{16}$$

$$\dot{Q} - \mathbb{P} = \frac{d}{dt}\int_{syst} e \, \rho \, dV \tag{17}$$

The idea that energy cannot be created or destroyed within a system, but that it can be exchanged is an old one and is easily accepted due to the fact that it has never been observed to be violated in nature.

The problem again is to extend the Energy Equation to cases where it is difficult to follow the control mass.

D. SUMMARY

The expressions for the FUPLAs for a system is summarized in Tables II and III.

Table II will be extended to other Laws which has not yet been discussed.

In Table III there are some symbols not defined so far. Refer to "Extension of the Control Volume Approach to other Fundamental Physical Laws" for explanation (Chapter IV).

TABLE II

Fundamental Physical Law	Name Usually Given to the Equation	Expression for a System
Conservation of Mass	Continuity Equation	$\frac{d}{dt}\int_{syst} \rho dV = 0$
Newton's Second Law of Motion	Momentum Equation or Momentum Theorem	$\frac{d}{dt}\int_{syst} \vec{v}\rho dV = \vec{F}_{net}$
First Law of Thermodynamics	Energy Equation	$\frac{d}{dt}\int_{syst} e\rho dV = \dot{Q} - \dot{P}$

TABLE III

Fundamental Physical Law	Name Usually Given to the Equation	Expression for a System
Angular Momentum	Moment of Momentum	$\frac{d}{dt}\int_{syst} (\vec{r} \times \vec{v})\rho dV = \vec{M}$
Second Law of Thermodynamics	Entropy	$\frac{d}{dt}\int_{syst} s\ dV \geq \int_{syst} \frac{\dot{q}}{T}\ dA$

Looking at the two previous tables, a clear pattern is seen in the form of the expressions for the FUPLAs. We may generalize this form as

$$\frac{d}{dt}\int_{syst} X\ dV = Y \tag{18}$$

X represents an extensive property that is "conserved" within the system <u>provided</u> that the right-hand side of Eq. 18 is zero (Y=0).

Y is a quantity that represents how the system interacts with the surrounding. Y's act on the system to change the X's.

Note that there has been no difference in our description for a system composed of a differential quantity of

fluid or, say, a solid such as a cannon ball. We find the ŝystem description, however, inconvenient in a flow situation because it is difficult to visualize it.

Suppose you can identify a fixed quantity of air flowing into a fan jet aircraft engine. It would be very difficult to follow that identified mass of air as it travels through the engine and experiences changes in properties. See Figure 5 below.

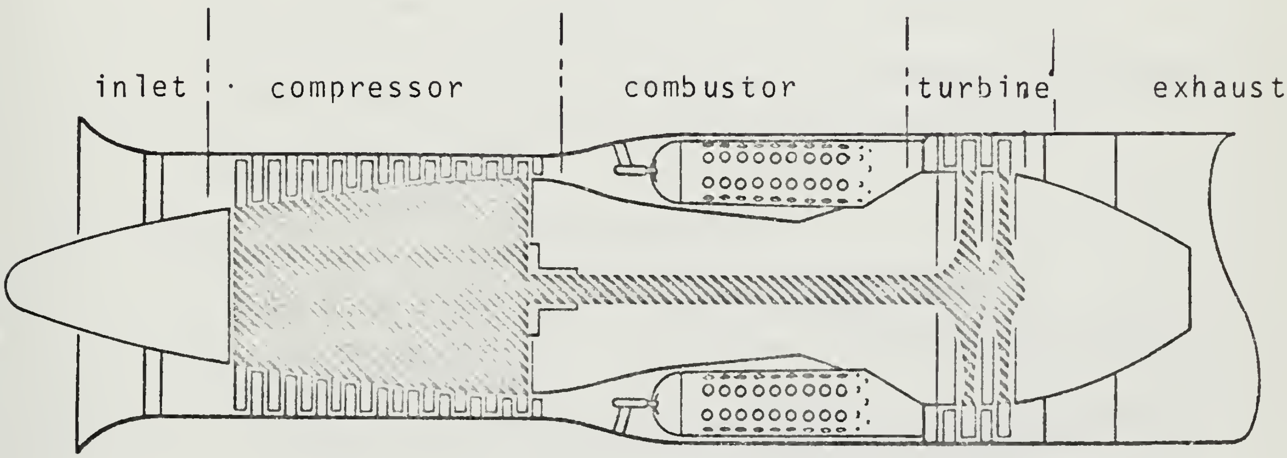

Figure 5. A jet engine

It would be much easier to choose a convenient ĉontrol v̂olume, fixed with respect to the engine for example and then investigate the changes in the air properties as it flows in and out of the ĉontrol v̂olume.

Exercise:

What would be a convenient ĉontrol v̂olume to describe what happens to air as it passes through a smoking pipe?

E. SELF-CHECK QUIZ NO. 2

Select the proper answer

1. A ŝystem is:

   a. the quantity of matter inside the ĉontrol v̂olume.

   b. a collection of matter of fixed identity.

   c. a collection of mass of fixed position.

   d. the set of substances that makes up a mixture.

2. The Momentum Equation is:

   a. the expression of Newton's Third Law of Motion.

   b. the mathematical expression of a Law based on observation of nature.

   c. other name given to the Moment of Momentum Theorem.

   d. the expression of the Law of Conservation of Angular Momentum.

3. The First Law of Thermodynamics:

   a. cannot be applied to a flow situation.

   b. has a mathematical expression usually called the Entropy Equation.

   c. is applicable to the interchange of heat and work between the ŝystem and the ĉontrol v̂olume.

   d. has a mathematical expression called the Energy Equation.

4. The expressions for the F̂undamental P̂hysical L̂aws for a ŝystem:

   a. show a clear pattern that makes possible to generalize them using a simple notation.

   b. are difficult to apply in certain flow situations.

   c. involves vector and/or scalar quantities.

   d. all the above are correct.

Answers: 1(b), 2(b), 3(d), 4(d).

5. In the generalized expression for the $\hat{\text{F}}$undamental $\hat{\text{P}}$hysical $\hat{\text{L}}$aws for the $\hat{\text{s}}$ystem the element of volume dV is related to:

a. the $\hat{\text{C}}$ontrol $\hat{\text{V}}$olume.

b. a volume inside which properties change from point to point.

c. the space occupied by the $\hat{\text{s}}$ystem which is stationary.

d. none of the above is correct.

Answer: 5(d)

III. DEVELOPMENT OF EXPRESSIONS FOR THE FUNDAMENTAL PHYSICAL LAWS USING THE CONTROL VOLUME APPROACH

Read continuously up to the end of Section B, but only if you are acquainted with the material up to here. Otherwise, do not proceed until understanding the system approach. You can expect the reading of this unit to take about twice as long as each of the previous units.

A. THE DEVELOPMENT IN GENERALIZED NOTATION

Instead of developing separate expressions for every Fundamental Physical Law, all of them can be done at once, using the generalized notation

$$\frac{d}{dt}\int_{syst} X\, dV = Y \tag{18}$$

Remember that in the above expression:

1. Is valid for a ______________________ only.
2. XdV is an extensive property--a ____ dependent quantity that is conserved ______ the system when Y is zero (except for the entropy).
3. Y is a quantity that affects the state of the system.
4. The differential dV refers to the volume filled by the ______________________ DO NOT CONFUSE WITH THE RIGOROUSLY AND COMPLETELY DIFFERENT CONTROL VOLUME.

We will use a control volume (CV) fixed with respect to ___________________________. Fluid can flow through this control volume. A typical CV is depicted in Fig. 6.

Answers: 1. system or control mass
2. mass
within
4. control mass or system
inertial coordinate system

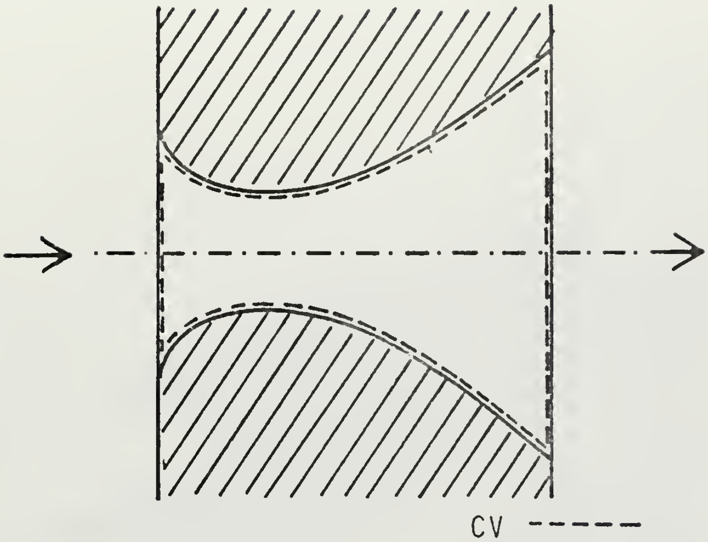

Figure 6. A Nozzle as an Example of Control Volume

It is possible to proceed with our development using the above shown $\hat{c}$ontrol $\hat{v}$olume, but a more general form or shape may be preferred, since we are trying to be general in the notation. In solving particular problems using the $\hat{c}$ontrol $\hat{v}$olume approach a wise choice of a convenient $\hat{c}$ontrol $\hat{v}$olume, suitable to particular situations, has to be made.

Let us consider the following general flow:

At time "t" the control volume is entirely occupied by the system ($V_{syst} = V_{CV}$).

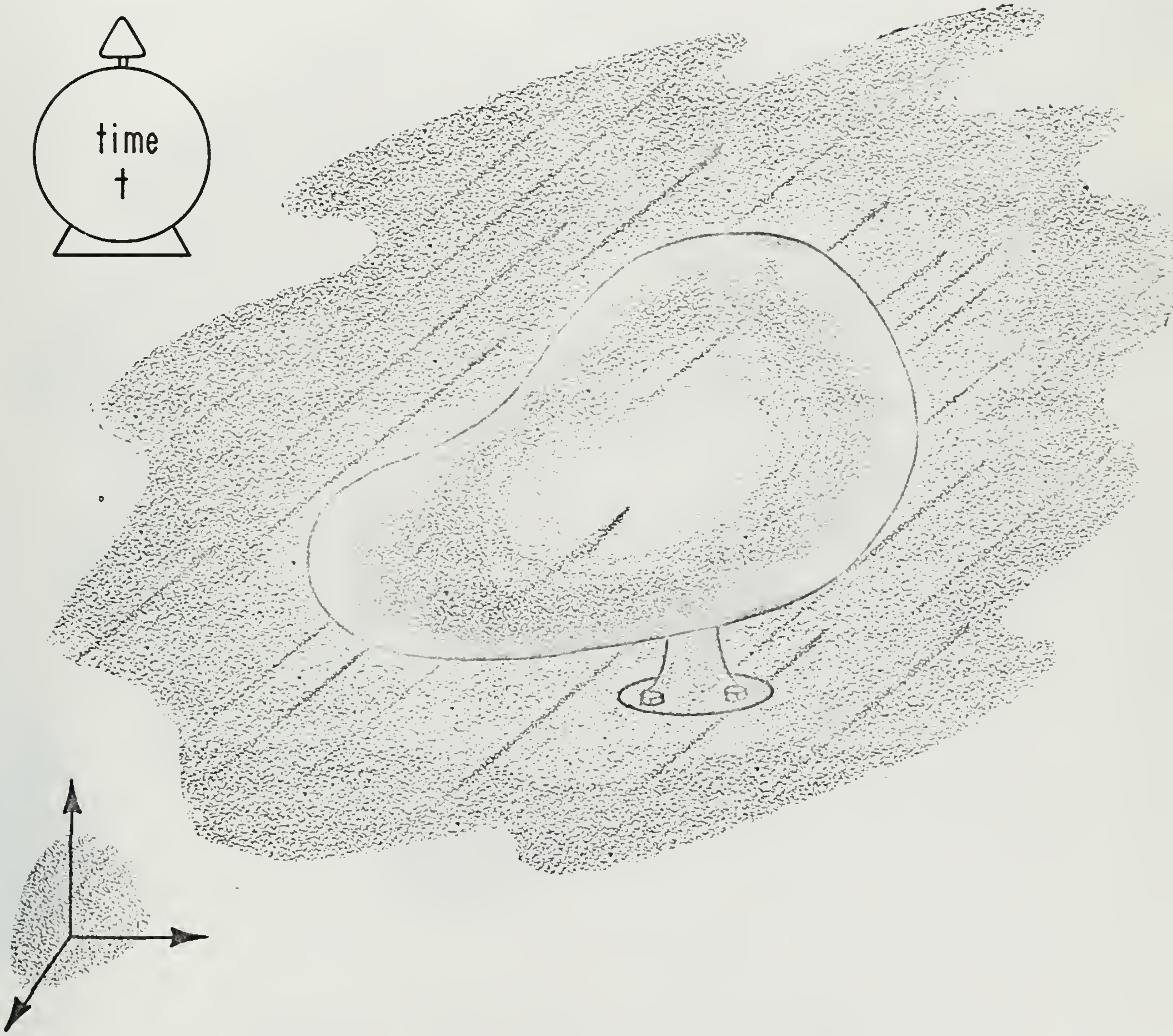

Figure 7. Snapshot at time "t"

At time "t + Δt" we can distinguish three different definite regions:

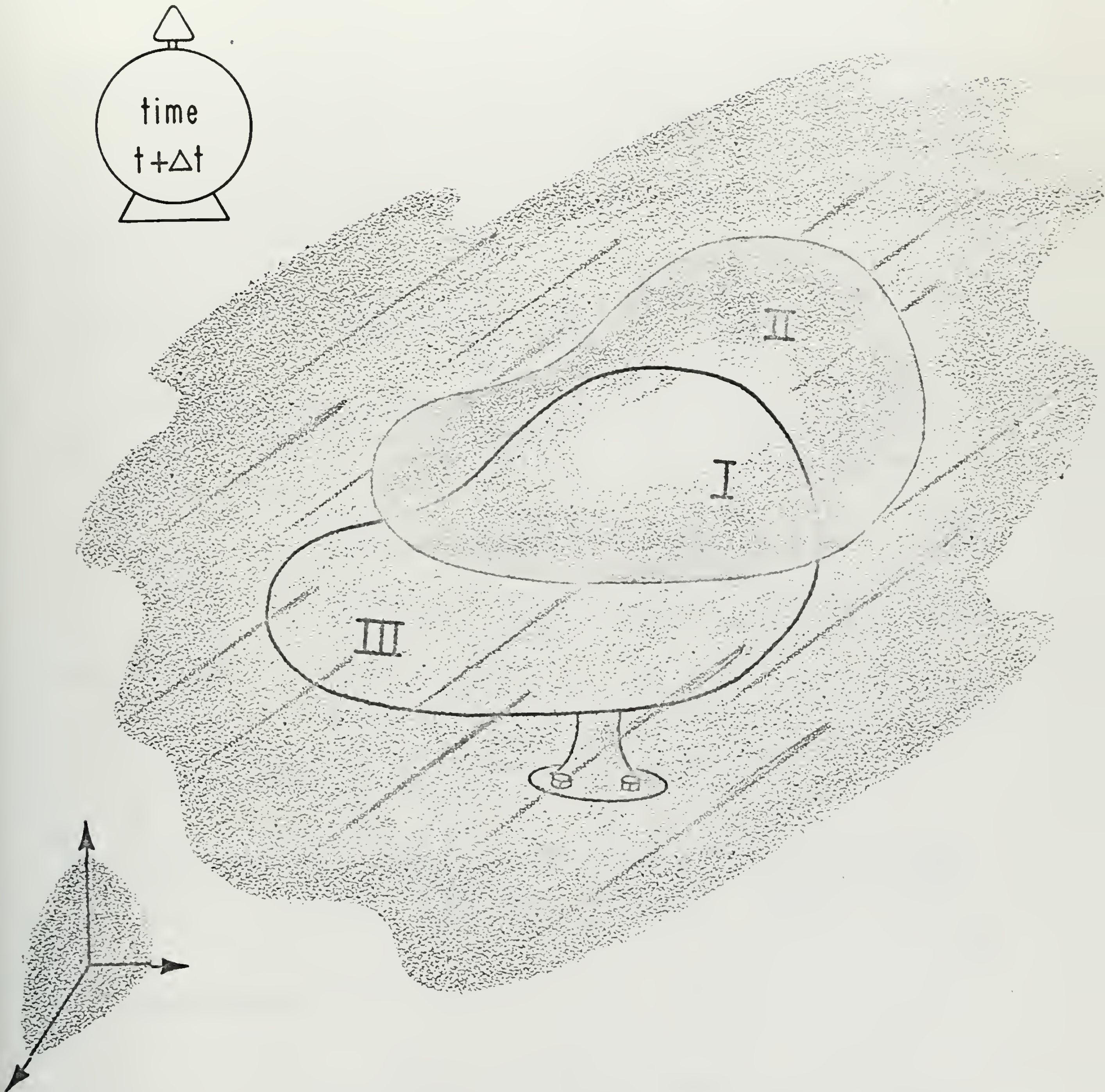

Figure 8. Snapshot at time "t + Δt"

Note that the $\hat{s}$ystem may have changed shape at time t + Δt.

Region I: common to the $\hat{c}$ontrol $\hat{v}$olume and the $\hat{s}$ystem

Region II: occupied by the portion of the $\hat{s}$ystem that left the $\hat{c}$ontrol $\hat{v}$olume during the interval Δt

Region III: portion of $\hat{c}$ontrol $\hat{v}$olume unoccupied by matter belonging to the $\hat{s}$ystem because it changed position during the interval Δt

From calculus, the time rate of change of the $\hat{e}$xtensive $\hat{p}$roperty X in the $\hat{s}$ystem is

$$\frac{d}{dt}\int_{syst} X \, dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{syst \\ @t+\Delta t}} X \, dV - \int_{\substack{syst \\ @t}} X \, dV\right) \tag{19}$$

Remember that in this case the differential dV refers to the volume occupied by the system not the control volume

Now at time $t + \Delta t$

$$\text{syst} = \text{region II} + \text{region I} \tag{20}$$

or

$$\text{syst} = \text{region II} + \text{CV} - \text{region III} \tag{21}$$

and at time t

$$\text{syst} = \text{CV} \tag{22}$$

Using these last volumetric considerations:

$$\frac{d}{dt}\int_{syst} X \, dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{RII \\ @t+\Delta t}} X \, dV + \int_{\substack{CV \\ @t+\Delta t}} X \, dV - \int_{\substack{RIII \\ @t+\Delta t}} X \, dV - \int_{\substack{CV \\ @t}} X \, dV\right) \tag{23}$$

Rearranging:

$$\frac{d}{dt}\int_{syst} X \, dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{CV \\ @t+\Delta t}} X \, dV - \int_{\substack{CV \\ @t}} X \, dV + \int_{\substack{RII \\ @t+\Delta t}} X \, dV - \int_{\substack{RIII \\ @t+\Delta t}} X \, dV\right) \tag{24}$$

Let us work with the right-hand side of Eq. 24 taking two terms at a time.

The first two terms represent changes with respect to time of the extensive property X inside the control volume.

Thus using the definition of limit:

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\int_{CV} X\ dV - \int_{CV} X\ dV\right) = \frac{\partial}{\partial t}\int_{CV} X\ dV \qquad (25)$$

Note that the right-hand side of Eq. 25 has a partial derivative, meaning that there is a change with respect to time of a quantity that depends on several independent variables. In the control volume $X = X(x,y,z,t)$.

We will take care now of the last two terms of Eq. 24

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\int_{\substack{RII\\ @\ t+\Delta t}} X\ dV - \int_{\substack{RIII\\ @\ t+\Delta t}} X\ dV\right)$$

These terms, associated with regions II and III, represent the amount of the extensive property X inside these regions at time $t+\Delta t$.

The amount of the extensive property X inside II at time $t+\Delta t$ can be related to the amount of X that can enter during Δt to this region, crossing that part of the control surface represented by segment 3-4 in Fig. 9a.

Similarly, the amount of property X inside III at time $t+\Delta t$ can be related to the amount of X that can enter to this region during the interval Δt crossing that part of the control surface represented by segment 1-2 in Fig. 9a.

Later we will find what this relation is, in order to transform the two last terms in Eq. 24.

Referring now to Fig. 9b: Identify the system drawing a segmented line as in Fig. 9b. Also identify the segments 1-2 and 3-4 of Fig. 9a. in the blank circles in Fig. 9b. (Answer in Fig. B2)

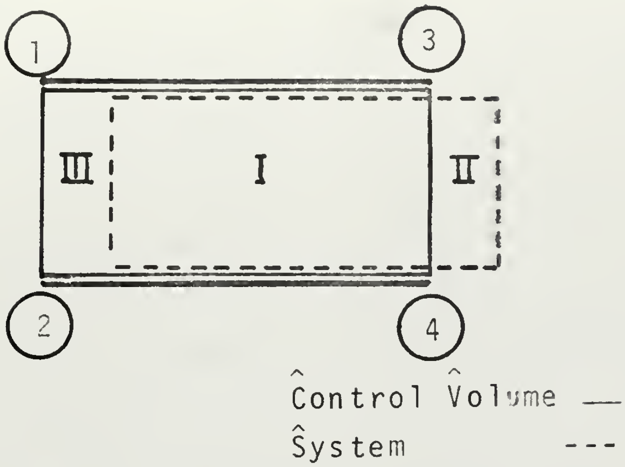

Figure 9a. One dimensional version of Figure 9b

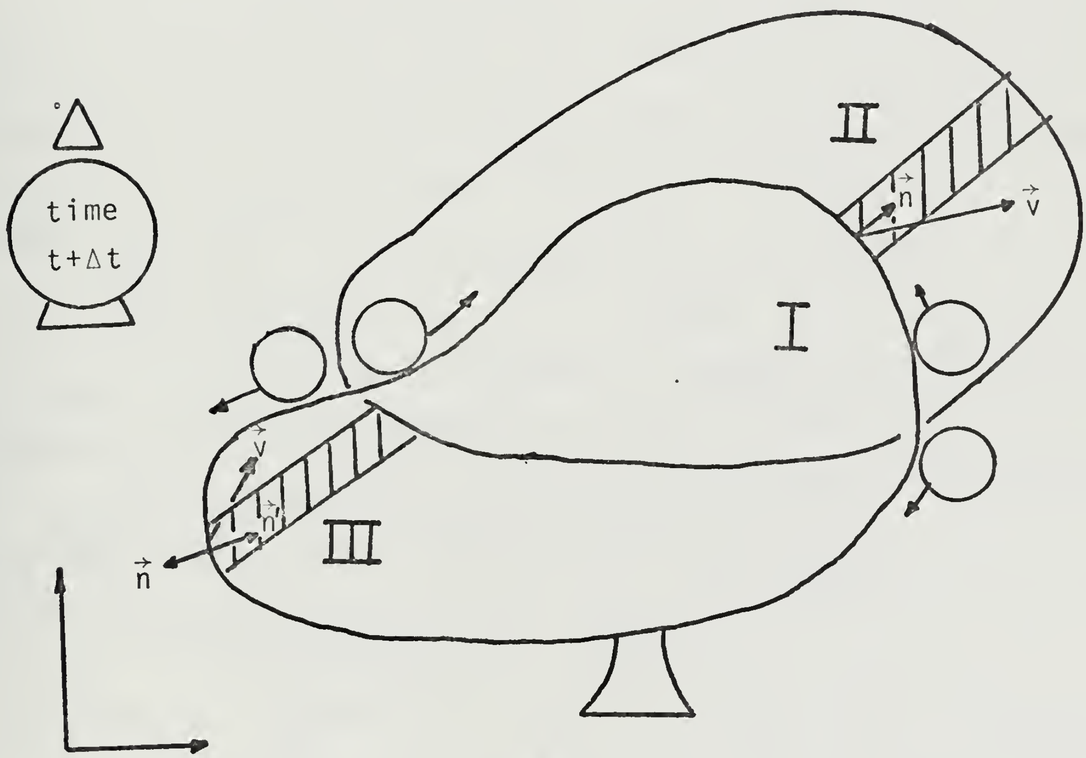

Figure 9b. Two dimensional projection

Let us find the mathematical expression of the amounts of property X flowing in and out of the control volume across the control surface. If dA is an increment in the control surface represented by segments 1-2 and 3-4 in Fig. 9, then

$\vec{v} \cdot \vec{n}$ is the component of $\vec{v}$ perpendicular to dA,
$(\vec{v} \cdot \vec{n})dA$ is the volumetric flow rate and
$X(\vec{v} \cdot \vec{n})dA$ is the flow of property X

Note that $X(\vec{v} \cdot \vec{n})dA\Delta t$ = amount of X that crosses dA in time Δt

Since $dV = dA(\vec{v} \cdot \vec{n})\Delta t$

Replacing this last expression for dV in the last term of the right-hand side of Eq. 24

$$\lim_{t \to 0} \frac{1}{\Delta t} \int_{\substack{RII \\ @t+\Delta t}} X \, dV = \lim_{t \to 0} \frac{1}{\Delta t} \int_{\substack{RII \\ @t+\Delta t}} X(\vec{v} \cdot \vec{n})dA \, \Delta t$$

The integral in the right-hand side of the above equation represents the extensive property X that flowed into region II during the interval Δt and it was said before that this flow is across that part of the control surface represented by 3-4 in Fig. 9. Then defining A_{out}, the area corresponding to this segment, one may write

$$\lim_{t \to 0} \frac{1}{\Delta t} \int_{\substack{RII \\ @\, t+\Delta t}} X(\vec{v} \cdot \vec{n}) \, dA\Delta t = \int_{A_{out}} X(\vec{v} \cdot \vec{n}) \, dA \tag{26}$$

Now if a vector $\vec{n}'$ is defined as <u>inward</u> normal (inward and outward refer to the control surface) the quantity

$A_{in} X(\vec{v} \cdot \vec{n}')dA\Delta t$ will represent the amount of property X that has flowed into region III during Δt, so

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\substack{RIII \\ @t+\Delta t}} X \, dV = \int_{A_{in}} X(\vec{v} \cdot \vec{n}')dA \tag{27}$$

and from Eqs. 26 and 27

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \left(\int_{\substack{RII \\ @\, t+\Delta t}} X \, dV - \int_{\substack{RIII \\ @\, t+\Delta t}} X \, dV \right) = \int_{A_{out}} X(\vec{v} \cdot \vec{n})dA - \int_{A_{in}} X(\vec{v} \cdot \vec{n}')dA \tag{28}$$

but $\vec{n} = -\vec{n}'$ (by construction)

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{RII \\ @t+\Delta t}} X\, dV - \int_{\substack{RIII \\ @t+\Delta t}} X\, dV\right) = \int_{A_{out}} X(\vec{v} \cdot \vec{n})\, dA + \int_{A_{in}} X(\vec{v} \cdot \vec{n})\, dA \quad (29)$$

The combination of the two terms on the right-hand side of Eqs. 28 and 29 represents the flow across the boundaries of the $\hat{c}$ontrol $\hat{v}$olume that occupies regions III and I at time t + Δt. We can rewrite Eq. 29 as

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{RII \\ @t+\Delta t}} X\, dV - \int_{\substack{RIII \\ @t+\Delta t}} X\, dV\right) = \oint_{cs} X(\vec{v} \cdot \vec{n})\, dA \quad (30)$$

Because the integration over Ain and Aout is the same as the integral over the whole $\hat{c}$ontrol $\hat{s}$urface and $\vec{n}ds \equiv d\vec{A}$ we can substitute Eq. 30 into Eq. 25

$$\boxed{\frac{d}{dt}\int_{syst} X\, dV = \frac{\partial}{\partial t}\int_{CV} X\, dV + \oint_{CS} X(\vec{v} \cdot d\vec{A})} \quad (31)$$

which can also be written as

$$\frac{d}{dt}\int_{syst} X\, dV = \int_{CV} \frac{\partial X}{\partial t}\, dV + \oint_{CS} X(\vec{v} \cdot d\vec{A}) \quad (32)$$

(Through the use of Leibnitz rule which is applied to the $\hat{c}$ontrol $\hat{v}$olume in space.)

The first term of the right-hand side may be considered as the rate of change of the quantity of X stored in the $\hat{c}$ontrol $\hat{v}$olume while the other term as the net flux of X out of the $\hat{c}$ontrol $\hat{s}$urface.

Equations 31 and 32 are very important because they relate the time rate of change of property X within a system or control mass with changes happening in the control volume.

Let us examine the first term of the right-hand side of Eq. 31. It represents the time rate of change of the extensive property X within the control volume. It is a triple integral or a volume integral, so when we write

$$\frac{\partial}{\partial t}\int_{CV} X \, dV \quad \text{we mean} \quad \frac{\partial}{\partial t}\iiint_{CV} X \, dV$$

This term is often called "local time rate of change" or storage term.

The second term of the right-hand side of Eq. 31 represents the net efflux (outflow) of the extensive property X across the boundaries of the control volume, also called control surface. The vector $d\vec{A}$ is perpendicular to the control surface at every point of it and is positive when directed outward. The dot product $\vec{v}$ times $d\vec{A}$ gives the projection of $\hat{v}$ over the normal to the surface direction. Again:

$$\oint_{CS} X(\vec{v} \cdot d\vec{A}) \quad \text{we mean} \quad \oiint_{CS} X(\vec{v} \cdot d\vec{A})$$

This term is often called "the convective term."

Equations 31 and 32 are transformation equations that enable us to obtain an expression for the FUPLAs for a control volume from the corresponding equation for a system.

From Eqs. 31 and 18 we obtain

$$\boxed{\frac{\partial}{\partial t}\int_{CV} X \, dV + \oint_{CS} X \, (\vec{v} \cdot d\vec{A}) = Y} \tag{33}$$

This is the generalized equation for the FUPLAs in control volume form.

B. PARTICULARIZATION OF THE GENERAL EXPRESSION

Now that Eq. 33 gives a general expression for the FUPLAs in terms of X and Y, it may be applied to specific cases:

1. Continuity Equation

$X = \rho$

$Y = 0$

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \oint_{CS}\rho\, \vec{v}\cdot d\vec{A} = 0 \qquad (34)$$

or

$$\boxed{\int_{CV}\frac{\partial \rho}{\partial t}\, dV + \oint_{CS}\rho\, \vec{v}\cdot d\vec{A} = 0} \qquad (35)$$

2. Momentum Equation

Note that here both X and Y must be vectors.

$\vec{X} = \rho\vec{v}$

$\vec{Y} = \vec{F}_{net}$

$$\frac{\partial}{\partial t}\int_{CV}\rho\vec{v}\, dV + \oint_{CS}\rho\vec{v}(\vec{v}\cdot d\vec{A}) = \vec{F}_{net}$$

or

$$\boxed{\int_{CV}\frac{\partial(\rho\vec{v})}{\partial t}\, dV + \oint_{CS}\rho\vec{v}(\vec{v}\cdot d\vec{A}) = \vec{F}_{net}} \qquad (36)$$

3. Energy Equation

$X = \rho e$

$Y = \dot{Q} - \mathbb{P}$

$$\frac{\partial}{\partial t}\int_{CV}(\rho e)\, dV + \oint_{CS}\rho e(\vec{v}\cdot d\vec{A}) = \dot{Q} - \mathbb{P} \qquad (37)$$

or

$$\boxed{\int_{CV} \frac{\partial(\rho e)}{\partial t}\, dV + \oint_{CS} \rho e\, (\vec{v} \cdot d\vec{A}) = \dot{Q} - \mathbb{P}} \qquad (38)$$

In supplement B, Eq. 38 is transformed into a more convenient form by introducing the enthalpy. This form is shown below (read supplement B to see how this was done).

$$\dot{Q} - \mathbb{P}_{CV} = \int_{CV} \frac{\partial}{\partial t}\, (\rho e)\; dV + \int_{CS} (h + \frac{|\vec{v}|^2}{2g_c} + \frac{g}{g_c} z)\, \rho(\vec{v} \cdot d\vec{A}) \qquad (39)$$

Where $\mathbb{P}_{CV}$ is the net useful (shaft) work.

C. SELF-CHECK QUIZ NO. 3

Select the proper answer

1. In the generalized notation expression for the FUPLAs

$$\frac{d}{dt}\int_{syst} X\,dV = Y$$

The quantity X:

a. must be a scalar.
b. is the unknown of the problem.
c. represents an extensive property.
d. none of the above.

2. At time "t" the control volume and the system:

a. are held stationary with respect to an inertial frame of reference.
b. occupies momentarily the same space.
c. are partially contained one inside the other.
d. cannot exist at the same time.

3. Which of the following expressions is correct based on calculus (noting that in the CV, X = X(x,y,z,t)):

a. $$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\substack{syst\\ @t}} X\,dV = \frac{d}{dt}\int_{CV} X\,dV$$

b. $$\lim_{\Delta t \to \infty} \frac{1}{\Delta t}\left(\int_{\substack{syst\\ @t}} X\,dV - \int_{\substack{syst\\ @t+\Delta t}} X\,dV \right) = \frac{d}{dt}\int_{CV} X\,dV$$

c. $$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{syst\\ @t+\Delta t}} X\,dV - \int_{\substack{syst\\ @t}} X\,dV \right) = \frac{\partial}{\partial t}\int_{CV} X\,dV$$

d. $$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{\substack{syst\\ @t+\Delta t}} X\,dV - \int_{\substack{syst\\ @t}} X\,dV \right) = \frac{d}{dt}\int_{CV} X\,dV$$

Answers: 1(c), 2(b), 3(c).

4. The following equation relates the expression for FUPLAs for a system with an expression valid for the control volume

$$\frac{d}{dt}\int_{syst} X dV = \frac{\partial}{\partial t}\int_{CV} X dV + \oint_{CS} X(\vec{v} \cdot d\vec{A})$$

a. This equation is directly derived from observation of nature.

b. The first term of the right-hand side is really a volume integral and is known as the "convective term."

c. The second term of the right-hand side of the equation is a surface integral sometimes called the "convective term."

d. Can be applied only to unsteady flows.

Answer: 4(c).

IV. EXTENSION OF THE $\hat{C}$ONTROL $\hat{V}$OLUME APPROACH TO OTHER $\hat{F}$UNDAMENTAL $\hat{P}$HYSICAL $\hat{L}$AWS

The Law of Angular Momentum and the Second Law of Thermodynamics for a $\hat{s}$ystem were stated without any further considerations in Table III.

Now we will try to obtain expressions of these laws applicable to a $\hat{c}$ontrol $\hat{v}$olume.

A. ANGULAR MOMENTUM

From the development of the momentum theorem for a $\hat{s}$ystem in Table II or Eq. 12

$$\vec{F}_{net} = \frac{d}{dt} \int_{syst} \vec{v}\, \rho \, dV \tag{40}$$

Performing the cross product of both sides of the above equations with a vector $\vec{r}$ whose origin is fixed to the same inertial frame of reference relative to which $\vec{v}$ is measured, we can write

$$\vec{M}_{net} = \frac{d}{dt} \int_{syst} \vec{r} \times \rho \, \vec{v} \, dV \tag{41}$$

The quantity in the left-hand side of the above equation is the moment of the force $\vec{F}_{net}$ with respect to the origin of $\vec{r}$. We have assigned to the moment the symbol $\vec{M}_{net}$.

The integrand on the right-hand side of Eq. 41 is the cross product of the moment arm r and the momentum and defines a quantity called moment of momentum or angular momentum.

Using the generalized notation introduced previously:

$$\vec{X} = \rho(\vec{r} \times \vec{v}) \tag{42}$$

$$\vec{Y} = \vec{M}_{net} \tag{43}$$

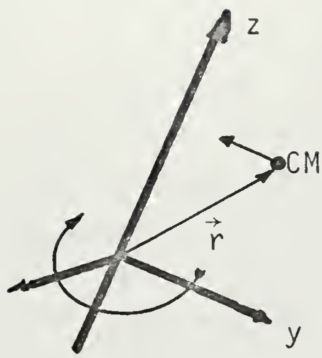

Figure 10. Coordinate System

Using the expression for the Fundamental Physical Laws developed for a control volume in Eq. 29:

$$\frac{d}{dt}\int_{syst}\rho(\vec{r} \times \vec{v})dV = \frac{\partial}{\partial t}\int_{CV}\rho(\vec{r} \times \vec{v})dV + \oint_{CS}\rho(\vec{r} \times \vec{v})(\vec{v}\cdot d\vec{A}) \tag{44}$$

So

$$\boxed{\vec{M}_{net} = \frac{\partial}{\partial t}\int_{CV}\rho(\vec{r} \times \vec{v})dV + \oint_{CS}(\vec{r} \times \vec{v})(\vec{v}\cdot d\vec{A})}$$

This is the so-called Moment of Momentum equation.

B. SECOND LAW OF THERMODYNAMICS

The Entropy Equation for a system as the mathematical expression of the Second Law of Thermodynamics was stated in Table III.

$$\frac{\delta Q}{T} \geq dS \tag{45}$$

inequality valid for an irreversible process

equality valid for a reversible process

Where δQ is a small quantity of heat transferred during an elementary part of the cycle, T is the absolute temperature at that point of the boundary, S the entropy, a property and s the specific entropy (entropy per unit mass)

$$S = \int_{syst} s\rho dV \tag{46}$$

Also

$$\delta Q = \int_{area} \dot{q} dt \, dA \tag{47}$$

where $\dot{q}$ is the heat flux (amount of heat flowing) per unit time per unit area

$$\dot{q} = \frac{d}{dA} \left(\frac{\delta Q}{dt}\right) \tag{48}$$

Dividing both sides of Eq. 47 by T

$$\frac{\delta Q}{T} = \int_{area} \frac{\dot{q}}{T} dt \, dA \tag{49}$$

Substituting Eq. 45 into Eq. 49

$$dS \geq \int_{area} \frac{\dot{q}}{T} dt \, dA \tag{50}$$

$$\frac{dS}{dt} \geq \int_{area} \frac{\dot{q}}{T} dA \tag{51}$$

but from Eq. 46

$$\frac{dS}{dt} = \frac{d}{dt} \int_{syst} s\rho \, dV \tag{52}$$

and finally from Eqs. 51 and 52

$$\frac{d}{dt} \int_{syst} s\rho dV \geq \int_{area} \frac{\dot{q}}{T} dA \tag{53}$$

C. SELF-CHECK QUIZ NO. 4

1. Can you associate Eq. 53 to the generalized notation that we had already developed?

2. What is the difference, besides the type of quantities involved, between Eq. 53 and the expressions for the other FUPLAs for a system in Table II?

3. Can you develop an expression for the Second Law of Thermodynamics valid for a control volume?

(See answers on next page)

Answers to Self-Check Quiz No. 4

1. $X = \rho s$

$$Y \geq \int_{\text{Area}} \frac{\dot{q}}{T}\, dA$$

2. The sign $\geq$ depends on what type of process is taking place for the heat being transferred in or out of the system. The equal sign holds if the process is reversible. The inequality sign holds if the process is irreversible.

3. Applying X and Y to Eq. 29

$$\boxed{\frac{\partial}{\partial t} \int_{CV} \rho s\, dV + \oint_{CS} \rho s(\vec{v} \cdot d\vec{A}) \geq \int_{\text{area}} \frac{\dot{Q}}{T}\, dA}$$

V. APPLICATION TO PARTICULAR PROBLEMS

A. CONTINUITY EQUATION

From Eq. 34.

$$\int_{CV} \frac{\partial \rho}{\partial t}\, dV + \oint_{CS} \rho(\vec{v} \cdot d\vec{A}) = 0$$

Applying this equation to the following case:

One-dimensional, steady, incompressible flow as shown in Fig. 11.

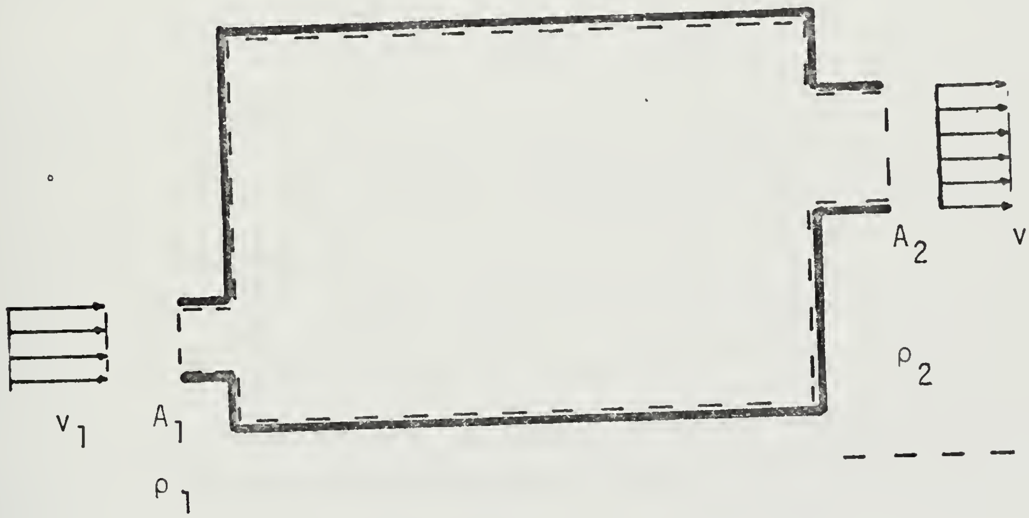

Figure 11. Container with unequal inlet and exit areas.

One-dimensional flow means that the velocity is a function of ____________________ coordinate. There is no change of velocity with respect to other coordinates.

Steady flow means that the velocity is not a function of

____________________.

Answers:
only one spatial
time

Incompressible flow means that the ________ does not change.

Considering the above, we can immediately make the first term of Eq. 34 equal to zero.

Now Eq. 34 becomes

$$\int_{CS} \rho \vec{v} \cdot d\vec{A} = 0 \qquad (53)$$

ρ is a constant so we can take it out of the integral in Eq. 54.

Also $\vec{v}$ is a function of x only so

$$\vec{v} = v_x \vec{i}$$

And

$$d\vec{A} = dA_x \vec{i} + dA_y \vec{j} + dA_z \vec{k}$$

But

$$\vec{v} \cdot d\vec{A} = v_x dA_x + 0\ dA_y + 0\ dA_z$$

So Eq. 54 becomes

$$\int_{CV} v_x dA_x = 0 \qquad (54)$$

It is only necessary to evaluate the integral Eq. 54 to stations 1 and 2 of the control volume since there is no flow through the walls

$$(\rho\ v_x A_x)_2 - (\rho\ v_x A_x)_1 = 0$$

Eliminating the subscript x: $v_{x_1} \equiv v_1$ and $v_{x_2} \equiv v_2$

$$v_1\ A_1 = v_2\ A_2 \qquad (55)$$

In the evaluation of the integral equation care must be taken with the signs remembering that dA_x can be positive or

Answer: density

negative, depending on the ______________________________
(i.e., at the inlet $\vec{V} \cdot d\vec{A}$ is a __________________ quantity).

In the following example we will show the system and control volume approach to an unsteady problem.

B. EXAMPLE OF APPLICATION OF THE ŜYSTEM AND ĈONTROL V̂OLUME APPROACH TO THE SOLUTION OF AN UNSTEADY PROBLEM

A partially evacuated bottle initially containing a mass m_1 is to be filled with air from the atmosphere until the mass reaches a value of m_2. Find the work to be done to achieve this and if there is no heat interaction, find the change in internal energy.

1. System Solution

A ŝystem or ĉontrol m̂ass is ______________________.

We have to choose a ŝystem consistent with the above definition. The mass during the process must remain unchanged. For this purpose we define the ŝystem boundaries so as to enclose the final mass m_2 at all times. (See Figures 12 and 13).

We are consistent with the definition because this ŝystem contains a fixed quantity of mass (m_2) although the volume that contains it changes.

The boundaries of the system at time t contain mass m_1 inside the bottle and mass $m_2 - m_1$ outside the bottle. The mass of fixed identity, i.e., the system is ______________________ (write an algebraic expression in terms of m_1 and m_2).

Answers:

the direction of the vector representing the area

negative

a quantity of mass of fixed identity

$m_2 = (m_2 - m_1) + m_1$

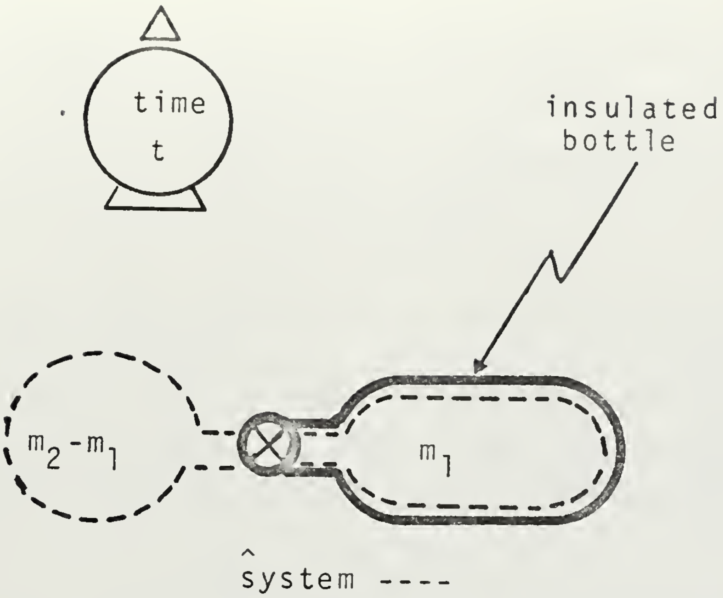

Figure 12. $\hat{S}$ystem solution. $\hat{S}$tate 1.

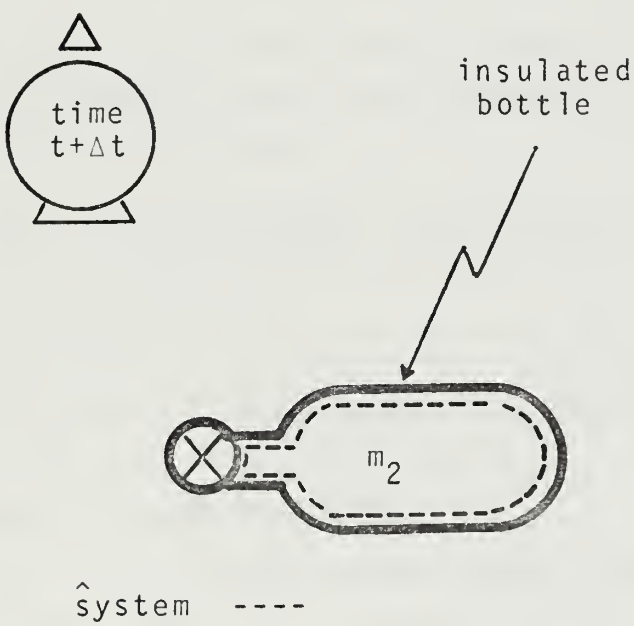

Figure 13. $\hat{S}$ystem solution. $\hat{S}$tate 2.

At time $t + \Delta t$ the boundaries of the system have changed forming a new volume, but being consistent with the definition of system, the mass contained in it is still ____.

Using the First Law of Thermodynamics with the sign convention of Fig. 4 we may write

$$ {}_1Q_2 - {}_1W_2 = E_2 - E_1 \tag{56}$$

The first term on the left-hand side of Eq. 56 may be eliminated because ______________________________.

If the only changes in energy are changes in internal energy, we can apply the above mentioned expression of the First Law of Thermodynamics for a system, reducing it to

$$ - {}_1W_2 = U_2 - U_1 $$

The change in internal energy is the internal energy at $t + \Delta t$ minus the internal energy at t

$$ \Delta U = m_2u_2 - [m_1u_1 + (m_2 - m_1)\, u_{atm}] \tag{57}$$

Where the subscript "atm" refers to the properties of the air at atmospheric conditions outside the bottle. The atmosphere acts as a reservoir (all properties remain constant).

The work ${}_1W_2$ is the work that has to be done on that part of the system outside of the bottle at time "t" to introduce it to the bottle.

$$ {}_1W_2 = p_{atm} \mathscr{v}_{atm}[0 - (m_2 - m_1)] \tag{58}$$

and Eq. 57 becomes

$$ p_{atm} \mathscr{v}_{atm}\,(m_2 - m_1) = m_2u_2 - [m_1u_1 - (m_2 - m_1)\, u_{atm}] \tag{59}$$

2. Control Volume Solution

If we now solve the same problem using the control volume technique, we have to choose the control volume conveniently.

Answers: m_2
the bottle is insulated

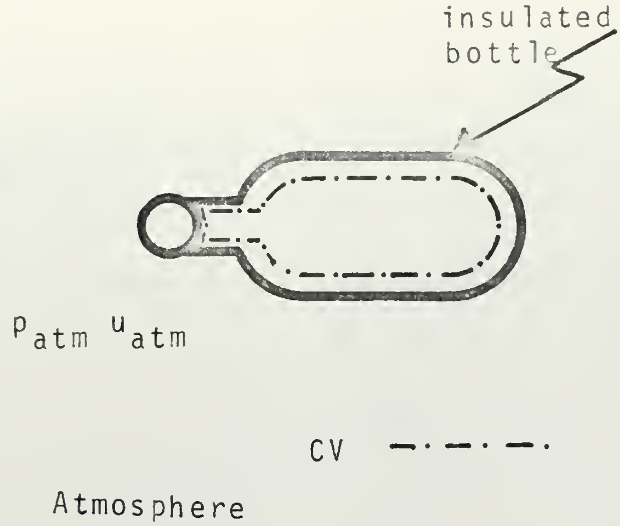

Figure 14. Ĉontrol v̂olume solution

During the process of filling the bottle the atmosphere is a ______. Its p̂roperties (i.e., p_{atm}, u_{atm} and h_{atm}) ____________________.

Recalling Eq. 39, the general expression for the Energy Equation for a control volume.

$$\dot{Q} - \mathring{\mathbb{P}}_{CV} = \int_{CV} \frac{\partial}{\partial t}(e\rho)\, dV + \oint_{CS} \left(h + \frac{|\vec{v}|^2}{2g_c} + z\,\frac{g}{g_c}\right)\rho(\vec{v}\cdot d\vec{A})$$

In the problem under consideration there is no heat transfer and we can neglect both $\mathbb{P}_{CV}$ and the potential energy changes. Thus Eq. 39 reduces to:

$$\int_{CV} \frac{\partial}{\partial t}(e\rho)\, dV + \oint_{CS} \left(h + \frac{|\vec{v}|^2}{2g_c}\right)\rho(\vec{v}\cdot d\vec{A}) = 0 \qquad (60)$$

We are solving a non-steady flow problem. Integrating Eq. 60 with respect to time, t_f being the time at which the bottle is filled with mass m_2, we have

Answers: reservoir
remain unchanged

$$\int_0^{t_f}\left[\int_{CV} \frac{\partial}{\partial t}(e\rho)\,dV + \oint_{CS}\left(h + \frac{|v|^2}{2g_c}\right)\rho(\vec{v}\cdot d\vec{A})\right]dt = 0 \quad (61)$$

Working the integral Eq. 61 by terms, we have for the first term

$$\int_0^{t_f}\left[\int_{CV} \frac{\partial}{\partial t}(e\rho)\,dV\right]dt = \int_1^2 dE \quad (62)$$

And if all changes in energy inside the control volume are changes in internal energy, Eq. 62 reduces to:

$$\int_0^{t_f}\left[\int_{CV} \frac{\partial}{\partial t}(e\rho)\,dV\right]dt = U_2 - U_1 \quad (63)$$

Where the subscripts 2 and 1 mean final and initial states respectively. Now let us define

$$h_t \equiv h + \frac{|\vec{v}|^2}{2g_c} \quad (64)$$

the total or stagnation enthalpy. Replacing Eq. 64 in the second term of Eq. 61 we obtain:

$$\int_0^{t_f}\left[\int_{CS}\left(h + \frac{v^2}{2g_c}\right)\rho(\vec{v}\cdot d\vec{A})\right]dt = \int_0^{t_f}\left[\int_{CS} h_t\,\rho(\vec{v}\cdot d\vec{A})\right]dt \quad (65)$$

But $\rho(\vec{v}\cdot d\vec{A}) = \frac{dm}{dt}$. For details on this see Fig. 9 and the development of Eq. 26. With this, Eq. 65 reduces to:

$$\int_0^{t_f}\left[\int_{CS}\left(h + \frac{|\vec{v}|^2}{2g_c}\right)\rho(\vec{v}\cdot d\vec{A})\right]dt = \int_0^{t_f}\left[\left(h_t \frac{dm}{dt}\right)_{out} - \left(h \frac{dm}{dt}\right)_{in}\right]dt \quad (66)$$

There is no outward flow of mass. The total inflow of mass during the interval from time 0 to t_f is $(m_2 - m_1)$. During this interval, the total or stagnation enthalpy of the air entering the control surface is equal to the constant atmospheric enthalpy (h_{atm}). With all these considerations, Eq. 66 reduces to:

$$\int_0^{t_f} \left[\int_{CS} \left(h + \frac{v}{2g_c}\right) \rho (\vec{v} \cdot d\vec{A}) \right] dt = -(m_2 - m_1)\, h_{atm} \qquad (67)$$

Substituting Eqs. 63 and 67 in Eq. 61 we obtain:

$$U_2 - U_1 = h_{atm}(m_2 - m_1)$$

And if

$$U_2 = u_2\, m_2$$

$$h_{atm} = p_{atm} \quad {}_{atm} + v_{atm}$$

$$U_1 = u_1\, m_1$$

$$p_{atm}\, v_{atm}(m_2 - m_1) = m_2 u_2 - \left[m_1 u_1 - (m_2 - m_1)\, u_{atm}\right] \qquad (68)$$

Equation 68 is exactly the same as Eq. 59 obtained using the system approach even though a few additional steps were needed.

VI. SUPPLEMENT A: BODY AND SURFACE FORCES

In discussing the Momentum Equation we had in Eq. 11

$$\vec{F}_{net} = \sum \vec{F}_{volume} + \sum \vec{F}_{surface}$$

And at that time the distinction between a body and a surface force was not explained in detail.

The body or volume forces are proportional to either the mass or volume of the body and comprise those forces involving action at a distance. These forces are a consequence of long range force fields, and include such examples as the force of gravitational attraction, magnetic forces and electrical (coulomb) forces. For accelerating coordinate systems, "inertia forces" such as centrifugal and coriolis forces would also be included. But in this case these last two forces must not be considered because our development was done for an inertial coordinate system.[2]

Surface forces are those forces which are exerted at the control surface by the material <u>outside</u> the ĉontrol v̂olume on the material inside the ĉontrol v̂olume. These forces are a consequence of short range force fields. Such forces are exerted in the form of surface stresses. We can distinguish two types of surface forces: (1) those arising from normal stresses, or pressures, acting on the ĉontrol ŝurface, and (2) those arising from shear stresses, or viscous stresses, acting on the ĉontrol ŝurface.

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[2] See footnote on page 22.

External Forces making up $\vec{F}_{net}$

- Body or volume forces
  - gravitational
  - magnetic
  - electrical
  - inertial
    - centrifugal
    - coriolis
- Surface forces
  - pressure → normal stress
  - viscous or shear → shear stress

VII. SUPPLEMENT B: INTERMEDIATE STEPS IN THE DERIVATION OF THE ENERGY EQUATION

Applying the general expression for the FUPLAs, Eq. 31, to the Energy Equation

$$\int_{CV} \frac{\partial(\rho e)}{\partial t}\, dV + \oint_{CS} \rho e(\vec{v}\cdot d\vec{A}) = \dot{Q} - \mathbb{P} \qquad (38)$$

Also stated without proof was that Eq. 38 can be transformed into Eq. 39. To do this, what $\mathbb{P}$ is must be considered. In Eq. 16, which is the most elementary expression for the First Law of Thermodynamics, was stated that $\mathbb{P}$ was the rate of work done by the $\hat{s}$ystem to the $\hat{s}$urroundings.

In Supplement A the nature of the forces that may be present and dividing them into body and surface forces, was examined. Body or volume forces do not appear in $\mathbb{P}$ since they do not represent work done on the $\hat{s}$urroundings. Rather, they appear in the energy term as a potential energy (for most cases of interest). We write e as:

$$e = u + \frac{|\vec{v}|^2}{2} + gz + \ldots$$

Hence, we restrict ourselves to the contribution of the <u>surface forces</u> to $\mathbb{P}$. These comprise normal and shear stresses and the objective here is to single out the contribution of the normal stresses which are assumed to be equal to the static pressure acting on the fluid.

Consider an element of fluid of mass ρdV entering the control volume.

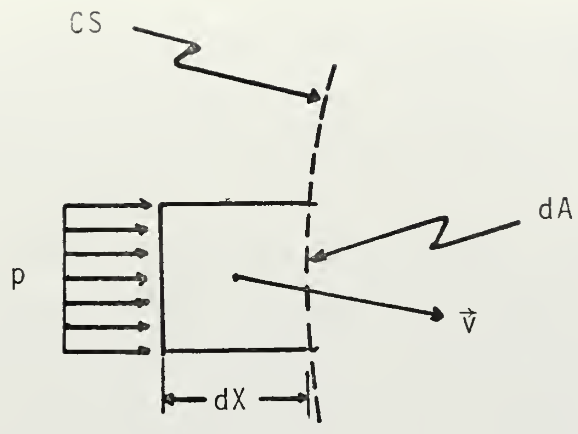

Figure B1. Diagram for calculating flow work

$$\text{work} = \text{force} \times \text{displacement}$$

$$= (p\,dA)\,dX \tag{B1}$$

$$\text{rate of work} = \frac{1}{\Delta t}(p\,dA\;dX) = \frac{1}{\Delta t}(p\,dV) \tag{B2}$$

Hence dividing by the mass (ρdV) we get

$$\text{rate of work/lbm} = \frac{1}{\Delta t}(p/\rho) = \frac{1}{\Delta t}(p\,\upsilon) \tag{B3}$$

The term[3] p/ρ is called flow work or flow energy since it represents the work to get the fluid into or out of the control volume. The important thing to note is that we can separate out the contribution of the normal stresses to $\mathbb{P}$ by simply accounting for the flow work. Since the fluid must always perform this work upon entering or leaving the control volume, this is a very useful step.

---

[3] $\upsilon \equiv 1/\rho$ is the specific volume (volume per unit mass, i.e., ft^3/Lb_m). Do not confuse this symbol υ with V for volume or $\vec{v}$ for velocity.

$$\mathbb{P} = \mathbb{P}_{CV} + \text{(rate of flow work)} \tag{B4}$$

In Eq. B4, $\mathbb{P}_{CV}$ represents what remains, namely, work done by shear stresses and shaft work. Since friction is usually handled indirectly, $\mathbb{P}_{CV}$ is commonly labeled as the net useful or shaft rate of work.

Let us see what the above means for a simple compressible substance. Suppose a fluid enters the $\hat{c}$ontrol $\hat{v}$olume at station (1) and exits at station (2) and suppose further that all the properties are homogeneous at each station. Now, on a per pound basis, the fluid must do an amount of work $p_1 \vartheta_1$ to enter the $\hat{c}$ontrol $\hat{v}$olume, and an amount of work $p_2 \vartheta_2$ to leave the $\hat{c}$ontrol $\hat{v}$olume. The work that may be performed is still

$$ {}_1W_2 = \int_1^2 p \, d\vartheta = -\int_1^2 \frac{p}{\rho^2} d\rho $$

so that analogous to Eq. B4,

$$ W_{CV} = \int_1^2 p \, d\vartheta_2 - (p_2 \vartheta_2 - p_1 \vartheta_1) $$

$$ = -\int_1^2 \vartheta dp = -\int_1^2 \frac{1}{\rho} \, dp $$

(this is an important result which is usually written down without proof. Show this to yourself starting with a steady flow form of the First Law and setting $\Delta KE = 0$ and $\Delta PE = 0$).

Now we are ready to arrive at the final form of the First Law namely, Eq. 39. Referring back to Fig. 9 we can do this.

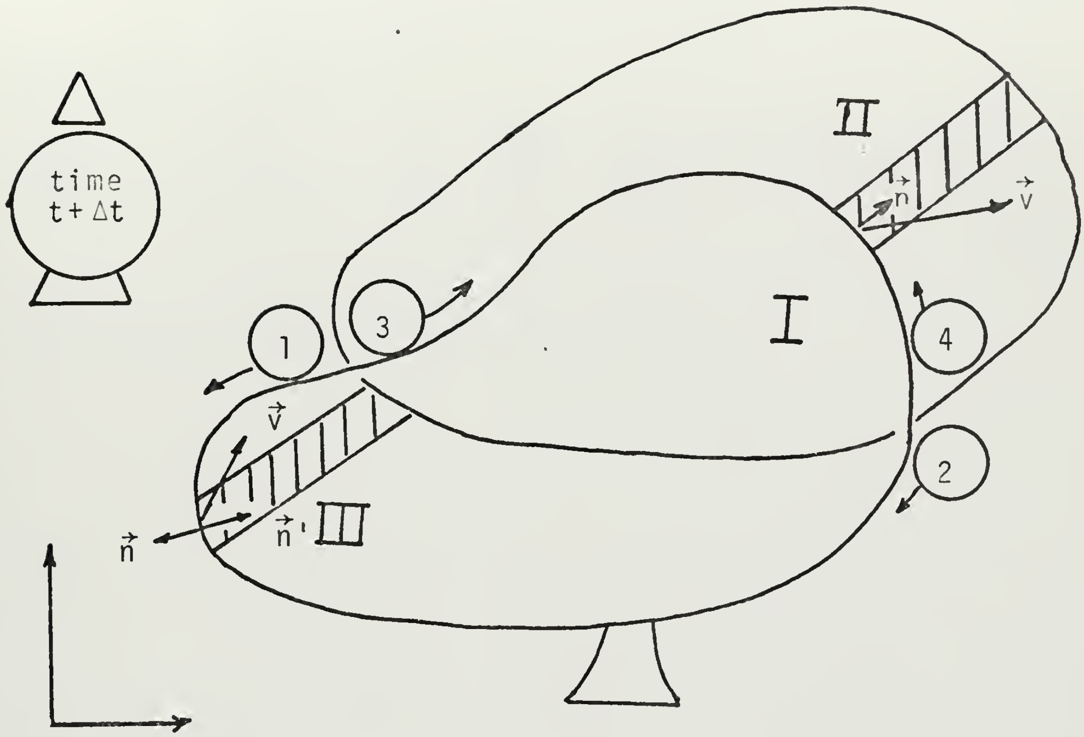

Figure B2. Two-Dimensional Projection at time t+Δt

Equation B2 can be written as

$$\frac{1}{\Delta t}\,(p\ dAdx) = \frac{1}{\Delta t}\,(p\ d\vec{A}\cdot\vec{v}\ \Delta t) \tag{B7}$$

Since for the fluid entering $d\vec{A}$ opposes $\vec{v}$ and for the fluid exiting, they are in the same direction, we get

$$\text{rate of flow work} = \lim_{\Delta t\to 0}\left[\frac{1}{\Delta t}\int_{Sout} p(\vec{v}\cdot d\vec{A}) + \frac{1}{\Delta t}\int_{Sin} p(\vec{v}\cdot d\vec{A})\right] \tag{B8}$$

$$= \oint_{CS} p(\vec{v}\cdot d\vec{A}) \tag{B9}$$

Hence, from Eq. B4

$$\mathbb{P} = \mathbb{P}_{CV} + \oint_{CS} p(\vec{v} \cdot d\vec{A}) \tag{B10}$$

And the First Law becomes

$$\int_{CV} \frac{\partial(\rho e)}{\partial t}\, dV + \oint_{CS} [u + \frac{p}{\rho} + \frac{|\vec{v}|^2}{2} + z\frac{g}{g_c}]\rho(\vec{v} \cdot d\vec{A}) = \dot{Q} - \mathbb{P}_{CV} \tag{B11}$$

Now substitute $h = u + \frac{p}{\rho}$ to get Eq. 39.

VIII. SUPPLEMENT C: EXPRESSIONS FOR THE FUNDAMENTAL PHYSICAL LAWS (FUPLAs) IN A DIFFERENTIAL FORM

The equations derived in the main text of the handout are in integral form. There are many applications where it is more useful to begin with a differential form of the equations.

An alternate for Eq. 34, the general expression for the FUPLAs is

$$\int_{CV} \frac{\partial X}{\partial t}\, dV + \oint_{CS} X(\vec{v} \cdot d\vec{A}) = \int_{CV} y\, dV \qquad (C1)$$

$$\text{where} \quad Y = \int_{CV} y\, dV$$

provided Y is expressible on a per unit volume basis.

Reference 14 or any other Vector Analysis book explains in detail a very useful mathematical tool, the Divergence Theorem.

The Divergence Theorem (also called Theorem of Gauss) states that, for any volume in a vector field $\vec{\zeta}$, the normal component $\vec{\zeta}\cdot\vec{n}$ integrated over the area is equal to the divergence of this vector field integrated over the volume. In mathematical symbols we have:

$$\oint \vec{\zeta}\cdot\vec{n}\, dA = \int_{V} (\nabla\cdot\vec{\zeta})dV$$

Reference 3 states that $\vec{\zeta}$ may be generalized to any tensor field. Now let us apply the Divergence Theorem to the second term of Eq. C1,

$$\oint_{CS} X(\vec{v} \cdot d\vec{A}) = \int_{CV} \nabla\cdot(X\, \vec{v})dV \qquad (C3)$$

where it is not difficult to recognize that we have identified $\vec{\zeta}$ in Eq. C2 with $X\vec{v}$. Note that this quantity is a vector if X is a scalar or a tensor if X is a vector!

Replacing Eq. C3 in C1

$$\int_{CV} \frac{\partial}{\partial t} X \, dV + \int_{CV} \nabla \cdot X \, \vec{v} \, dV = \int_{CV} y \, dV \qquad (C4)$$

Therefore, since the control volume is arbitrary and common to all three terms

$$\frac{\partial X}{\partial t} + \nabla \cdot (X\vec{v}) = y \qquad (C5)$$

This is the differential form of the $\hat{F}$undamental $\hat{P}$hysical $\hat{L}$aws using the generalized notation X. We will apply Eq. C5 to entries in Table II.

A. CONTINUITY

$$X = \rho \quad \text{and} \quad y = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0} \qquad (C6)$$

For steady flow

$$\nabla \cdot \rho \vec{v} = 0 \qquad (C7)$$

and for incompressible[4] flow (ρ = ________).

Thus

$$\nabla \cdot \vec{v} = 0 \qquad (C8)$$

---

[4] Some books (like Ref. 4, p. 44) goes straight from Eq. C6 to C8 without assuming steady flow. ·This can be done if the flow is incompressible, since then ρ does not change either with time or position.

B. MOMENTUM

$\vec{X} = \rho\vec{v}$ and $y = \vec{f}_{net}$ ($\vec{F}_{net}$ per unit volume)

$$\frac{\partial(\rho\vec{v})}{\partial t} + \nabla\cdot(\rho\vec{v}\vec{v}) = \vec{f}_{net} \tag{C9}$$

Note that the product $\vec{v}\vec{v}$ is not defined in vector analysis. It is a dyadic product (a tensor) and we should be using tensor notation. However, $\nabla\cdot(\rho\vec{v}\vec{v})$ represents a vector [Ref. 15, p. 200].

The first term of Eq. C9 can be written as

$$\frac{\partial(\rho\vec{v})}{\partial t} = \vec{v}\frac{\partial\rho}{\partial t} + \rho\frac{\partial\vec{v}}{\partial t} \tag{C10}$$

and the second term can be written

$$\nabla\cdot(\rho\vec{v}\vec{v}) = \vec{v}(\nabla\cdot\rho\vec{v}) + (\rho\vec{v}\cdot\nabla)\vec{v} \tag{C11}$$

Replacing Eqs. C10 and C11 in Eq. C9

$$\vec{v}\left(\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{v}\right) + (\rho\vec{v}\cdot\nabla)\vec{v} + \rho\frac{\partial\vec{v}}{\partial t} = \vec{f}_{net} \tag{C12}$$

The parenthesis of the first term in Eq. C12 is zero from continuity Eq. C6, so Eq. C12 reduces to

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = \vec{f}_{net} \tag{C13}$$

There is a compact notation often used in connection to these types of equations

$$\frac{D[\ ]}{Dt} \equiv \frac{\partial[\ ]}{\partial t} + (\vec{v}\cdot\nabla)[\ ] \tag{C14}$$

$\frac{D[\ ]}{Dt}$ is called the "substantial," "total," or "flow" derivative according to different authors. Rewriting Eq. C13 using the notation introduced in C14

which is an equation of motion analogous to $m \frac{d\vec{v}}{dt} = \vec{F}_{net}$ the expression for Newton's second law for a solid, such as a cannon ball. A deeper insight is necessary regarding the term on the right-hand side of Eq. C15. This term is related to $\vec{F}_{net}$ whose meaning was explained in supplement A. So we can write

$$\vec{f}_{net} = \vec{f}_{volume} + \vec{f}_{surface} \tag{C16}$$

For the moment we will concentrate our discussion on $\vec{f}_{surface}$ since it is not immediately apparent how to express it on a per unit volume basis. The surface forces depend on the rate at which the fluid is strained by the velocity field. The system of forces determine a state of stress and the relationship between stress and strain can only be given empirically. Recalling again Supplement A, a distinction can be made between forces arising from normal stresses and forces arising from shear stresses.

$$\vec{f}_{surface} = \vec{f}_{normal} + \vec{f}_{shear} \tag{C17}$$

The force due to the normal stress may be written:

$$\vec{f}_{normal} = - \nabla p \tag{C18}$$

and the force due to the shear stress, under the Stokes' hypothesis,

$$\vec{f}_{shear} = \mu \nabla^2 \vec{v} \tag{C19}$$

Where μ is the viscosity of the fluid. It is a property of the fluid (and may be strongly dependent on temperature). Replacing Eqs. C19 and C18 in C17

$$\vec{f}_{surface} = - \nabla p + \mu \nabla^2 \vec{v} \tag{C20}$$

We have only shown between Eqs. C16 and C20 how the different kinds of stresses make up $\vec{f}_{surface}$. A complete derivation of Eq. C20 can be found in Ref. 4. In the derivation of Eq. C20 it was assumed that the fluid is isotropic, namely, the components of the stresses are equal in all direction and that it is Newtonian, i.e., a linear relation exists between

the rate of strain and the stress, the relation between the two being empirical.

Replacing Eq. C20 in C16

$$\vec{f}_{net} = \vec{f}_{volume} - \nabla p + \mu\nabla^2 \vec{v} \qquad (C21)$$

and replacing Eq. C21 in C15

$$\boxed{\rho\frac{D\vec{v}}{Dt} = \vec{f}_{volume} - \nabla p + \mu \nabla^2 \vec{v}} \qquad (C22)$$

Equation C22 is a very useful form of the equations of motion called the Navier-Stokes Equations (a set of three equations, one for each coordinate). They form the starting point for a course in boundary layer theory, Written as in Eq. C22 it has only the restrictions of isotropic, incompressible, Newtonian flows. The incompressible restriction arises from the fact that we had treated μ as a constant taking it out of the Laplacian in the second term of the right-hand side of Eq. C22. For compressible flow, where there exists gradients of temperature that affect strongly the value of μ, the above mentioned term should be written $\nabla\cdot\mu\nabla\vec{v}$. In the derivation of Eq. C22, the compact form of Navier-Stokes equations, most of the effort was given to explaining the significance of each term and its origin. For a complete and rigorous derivation of these equations we suggest the use of Ref. 4 in the context of an appropriate course.

In Aeronautical Engineering, the Navier-Stokes equations are useful in flows (not fluids) where the viscous effects are important, i.e., flows where large velocity gradient exist. Such situations are found in boundary layers where there is a change of velocity between a value of zero at the wall and a value different from zero at the free stream. The gradient creates important viscous forces taken into account in the last term of Eq. C22. In the free stream outside the boundary

layer, although μ (the viscosity) exists as in the boundary layer region (same fluid), the viscous forces do not exist because there is no velocity gradient, and the last term in Eq. C22 is zero.

In which other case would you do the same ? ______________

In these two cases Eq. C22 reduces to

$$\rho\frac{D\vec{v}}{Dt} = \vec{f}_{volume} - \nabla p \tag{C23}$$

Write Euler's Equation of Motion for one dimensional flow using the definition of total derivative

(C24)

For steady flow and neglecting the volume forces, Eq. C24 reduces to:

$$v\,\frac{dv}{dx} = -\frac{1}{\rho}\frac{dp}{dx} \tag{C25}$$

Why does Eq. C25 only contain total derivatives? ______ ______________________________.

Now integrating Eq. C25 we get

$$\frac{v^2}{2} + \int\frac{dp}{\rho} = \text{constant} \tag{C26}$$

Or if the flow is ___________________, the pressure term can be integrated to give

$$\frac{v^2}{2} + \frac{p}{\rho} = \text{constant} \tag{C27}$$

Answers:

when $\mu=0$ (inviscid fluid)

$\rho(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}) = \frac{\partial p}{\partial x} + f_{vol}$ x-dir

because v and p are functions of one coordinate (say x for example). One dimensional steady flow.

incompressible (ρ is constant)

This is called Bernoulli's Equation. Sometimes it has an additional term $z\frac{g}{g_c}$ coming from the body forces when we neglected in Eq. C24 (as they pertain to a gravity field).

This equation looks like an Energy Equation but it is not complete as we shall see in part C of this Supplement.

Exercise:

What does it mean when the flow is Newtonian?

On what property does the viscosity primarily depend?

It can be shown that the assumption of one-dimensional flow can be replaced by the restriction that the constant in Eq. C27 takes different values for different streamlines. In other words, the left-hand side of Eq. C27 is equal to a different constant, depending on which streamline of the flow is considered.

Also, it can be shown that Eq. C27 is valid everywhere (no matter if we change streamline) if the flow is irrotational ($\nabla \times \vec{v} = 0$).

Equation C27 is known as ______________ Equation. The assumptions and constraints in its derivation are: _________ ________________________________.

Answers: Bernoulli

Incompressible, steady, one-dimensional flow.

Inviscid fluid. Volume forces neglected.

C. ENERGY

In our development of the Energy Equation for a control volume, Eq. 38 we stated that this form was not very convenient and in Supplement B we transformed it into Eq. 39.

The differential form of the Energy Equation, Eqs. 38 or 39 can be derived in a manner similar to the Momentum Equation, thus only the results will be given here. For a description of the various terms and their application see Ref. 5.

The following form is usually quoted [Ref. 5] for the Energy Equation:

$$\rho\frac{\partial e}{\partial t} + \rho(\vec{v}\cdot\nabla)e - p\nabla\cdot\vec{v} = -\nabla\cdot\dot{\vec{q}} + \mu\Phi \qquad \text{(C29)}$$

or in terms of the enthalpy

$$\rho\frac{\partial h}{\partial t} + \rho(\vec{v}\cdot\nabla)h + [\frac{\partial p}{\partial t} + (\vec{v}\cdot\nabla)p] = -\nabla\cdot\dot{\vec{q}} + \mu\Phi \qquad \text{(C30)}$$

where $e = u$ (neglecting changes in kinetic and potential energy)

$$\dot{Q} = \oint_{CS} \dot{\vec{q}}\cdot d\vec{A} \qquad \text{(C31)}$$

and Φ = dissipation function acting through the fluid viscosity

If neglecting radiation heat transfer and assuming that Fourier's Law of Heat Conduction applies, then

$$\dot{\vec{q}} = -k\nabla T \qquad \text{(C32)}$$

and

$$\nabla\cdot\dot{\vec{q}} = -\nabla\cdot(k\nabla T) \qquad \text{(C33)}$$

$$\nabla\cdot\dot{\vec{q}} = -k\nabla^2 T \quad \text{for } k = \text{constant} \qquad \text{(C34)}$$

The solution to the equation above represents the effect of fluid motion on heat transfer and is studied under the title of "Convective Heat Transfer."

Substituting Eq. C34 into Eq. 31

$$\rho\frac{\partial h}{\partial t} + \rho(\vec{v}\cdot\nabla)h + [\frac{\partial p}{\partial t} + (\vec{v}\cdot\nabla)\ p] = k\nabla^2 T + \mu\Phi \tag{C35}$$

D. SELF-CHECK QUIZ NO. 5

1. As a consequence of steady flow

   a. the velocity is not a function of the position of the fluid particle.

   b. density is not a function of the velocity.

   c. velocity is not a function of time.

   d. density is constant.

2. What term(s) would you eliminate from Eq. C22 if you are told that the fluid/flow is

   a. Inviscid.

   b. Incompressible.

3. If for a fluid element it is found that there exists a linear relation between the rate of strain and stress, the fluid is said to be

   a. Eulerian.

   b. Newtonian.

   c. Lagrangian.

   d. Isotropic.

4. From the differential form of the Momentum Equation we derived several equations associated with great scientists and mathematicians of the past. The expressions known as Euler, Bernoulli and Navier-Stokes Equations have in common that they are:

   a. Energy Equations.

   b. Continuity Equations.

   c. Moment Equations.

   d. Equations of Motion.

Answers: 1. (c)
2. a. last of the right-hand side, b. None.
3. (b)
4. (d)

5. Match with the name of the equations (Column 1) derived in this supplement all the assumptions or constraints made in its derivation (Column 2).

Column 1	Column 2
a. Navier-Stokes Equations	a. Inviscid fluid.
b. Euler Equation	b. One-dimensional flow.
c. Bernoulli Equation	c. Isotropic fluid.
	d. Incompressible flow.
	e. Newtonian fluid.
	f. Steady flow.
	g. No velocity gradients.

Answer: 5. a: c, d, e.
b: a or g.
c: a or g, b, d, f.

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13. ABSTRACT

This thesis contains a handout covering the Fundamental Physical Laws (Continuity, Momentum, and Energy) used in Aeronautical Engineering which are transformed from the control mass or system form into the control volume form. It is intended that this handout serve as a self-studying guide for students in the core of the Aeronautical Engineering Program at the Naval Postgraduate School and as a reference during the graduate level courses.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
control volume						
control mass or system						
Fundamental Physical Laws						
Continuity						
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